

# CE 3501 - Design of Reinforced Cement

## Concrete Elements

### Methods of Design of Reinforced Concrete Structures

#### Unit - I . Introduction

#### Objective of Structural Design

\* Design the structure for stability, strength & serviceability.

\* Economical & aesthetic.

\* The design of the structure must satisfy three basic requirements

\* Stability → to prevent overturning/sliding/buckling under the action of load.

\* Strength → resist safely the stresses induced loads

\* Serviceability → stiffness, crack, deflection, durability (including corrosion resistance)

#### Steps in RCC Structural Design process:

\* The process of structural design involves the following stages.

\* Structural planning → Position & orientation of columns  
Positioning of beams  
Spanning of slabs  
Layout of stairs  
Selecting proper type of footing

\* Action of forces & Computation of loads

\* Methods of Analysis

\* Member Design

\* Detailing

\* Drawing

\* Preparation of Schedules.

## Types of loads on structures → IS: 456: 2000 ; P. NO. 433 ; Cl: 19 to 19.6

- \* Dead loads → IS: 875 Pt: 1 - 1987
- \* Live load (or) Imposed loads → IS: 875 ; Pt: 2 - 1987
- \* Wind loads → IS: 875 ; Pt: 3 - 1987
- \* Snow loads → IS: 875 ; Pt: 4 - 1987
- \* Earthquake loads → IS: 1893 - 2014
- \* Special loads → IS: 456-2000 ; Cl: 19.6

(i) Foundation movement → IS: 1904

(ii) Elastic axial shortening

(iii) Soil & fluid pressure → IS: 875 ; Pt: 5

(iv) Vibration

(v) Fatigue loads

(vi) Impact loads → IS: 875 ; Pt: 5

(vii) Erection loads → IS: 875 ; Pt: 2

(viii) Stress concentration effect due to point load

## Load combinations:- IS: 456: 2000 → P. NO: 68 ; TABLE: 18

DL + I.L

DL + WL (or) D.L + EQ. L

DL + I.L + WL (or) DL + I.L + E.L

Where,  
DL → Dead load  
WL → Wind load  
EL → Earthquake load  
I.L → Imposed (or) Live load

## code of practices & specifications:-

(i) IS: 456-2000 → plain & Reinforced Concrete code of Practice is an Indian standard code of practice for general use of plain & Reinforced concrete.

\* Latest revision of this standard was done in year 2000.

\* This code uses  $\sigma$  limit state design approach as well as the working <sup>stress</sup> design approach.

\* It is written for use in India.

\* It contains 5 sections and 8 annexes.

Section 1 - General

Section 2 - Materials, Workmanship, inspection & Testing

Section 3 - General Design considerations

Section 4 - special Design requirements for structural members & systems

Section 5 - structural Design (Limit state method)

## ii) SP:16-1980

\* The structural practice hand book SP:16-1980 - Design Aids for Reinforced Concrete to IS:456-1978 has tables & charts that help structural Engineers to design simple sections.

## Concept of working stress method:

\* It is assumed as elastic, steel and concrete act together elastically, where the relationship b/w loads & stresses is linear.

## Assumptions of WSM:

- \* plane section before bending will remain plain after bending.
- \* Bond b/w steel & concrete is perfect with in elastic limit of steel.
- \* The steel & concrete behaves as linear elastic material.
- \* All tensile stresses are taken by reinforcement and none by concrete

- \* The stresses in steel & concrete are related a factor known as modular ratio.
- \* The stress-strain relationship of steel and concrete is a straight line under working load.

### Limitations of WSM:-

- \* The assumptions of linear elastic behaviour and control of stresses within specially defined permissible stresses are unrealistic due to several reasons (i.e., creep, shrinkage and other long term effects, stress concentration and other secondary effects).
- \* Different types of load acting simultaneously have different degrees of uncertainties. This cannot be taken into account in the WSM.
- \* The actual FOS is not known in this method design.

### Limit state Method:-

- \* Limit states are the acceptable limits for the safety and serviceability requirements of the structure before failure occurs.
  - \* The design of structures by this method will thus ensure that they will not reach limit states and will not become unfit for the use for which they are intended.
- (i) Limit states of collapse  $\rightarrow$  It deals with strength and stability of the structure under maximum design load.
  - (ii) Limit states of serviceability  $\rightarrow$  It deals with deflection and cracking under service load, durability under working environment, stability, fire resistance etc.

## (ii) Load Factor (or) Ultimate load Method :-

- \* Ultimate or collapse load is used as design load.
- \* Ultimate loads are obtained by increasing the working/service loads suitably by some factors
- \* The factors are multiplied by the working loads to obtain ultimate loads are called as load factors.
- \* These load factors give the exact margins of safety in terms of loads.

## Advantages of Ultimate load Method :-

- \* The method is more realistic as compared to working stress method because ultimate load method takes into account the non-linear behaviour of the concrete.
- \* This method gives exact margin of safety in terms of load unlike working stress method which is based on the permissible stresses which do not give any idea about the failure/collapse load.
- \* The sections designed by ultimate load method are thinner and require less reinforcement.  
∴ This method is economical as compared to WSM.

## Limitations of Ultimate load Method :-

- \* This method gives very thin sections which leads to excessive deformations and cracking, thus making the structure unserviceable.
- \* No factors of safety are used for material stresses.

## Properties of concrete:

IS: 456: 2000 - P. No. 15

- \* Grades (M20, M25, M30 etc) → higher grades of concrete should be used for severe, <sup>or</sup> severe & extreme environments
- \* Compressive strength
- \* Characteristic strength →  $f_d = \frac{\text{characteristic strength}}{\text{partial safety factor}}$   
for concrete 1.5  
for steel 1.15
- \* Tensile strength →  $f_{ct} = 0.7 f_{ck}$   
it varies  $\frac{1}{8}$  to  $\frac{1}{12}$  of cube compressive strength.
- \* Durability
- \* Creep
- \* Shrinkage
- \* Unit weight → PCC = 24 kN/m<sup>3</sup>  
RCC = 25 kN/m<sup>3</sup>
- \* Modular ratio → Short term modular ratio =  $\frac{E_s}{E_c} = \frac{\text{modulus of Elasticity of steel}}{\text{Modulus of elasticity of concrete}}$
- \* Poisson's ratio → Long term modular ratio (m) =  $\frac{280}{30 f_{ck}}$   
For HSC = 0.1 to 0.2 for weak mixes  
Normally taken  $\mu = 0.15$  for strength design  
 $\mu = 0.2$  for serviceability criteria

## Properties of steel:-

- \* Hardness
- \* Toughness
- \* Tensile strength
- \* Yield strength
- \* elongation
- \* Fatigue strength
- \* Corrosion
- \* Plasticity
- \* Malleability
- \* Creep

## Materials

- \* Cement → binding material  
 \* sand (or) F.A → fills voids of C.A  
 PCC { Coarse Aggregate →  
 Water →

## RCC

Cement  
 F.A + Steel  
 C.A  
 Water

## Quality of concrete depends upon

Grade of ~~concrete~~ <sup>cement</sup>, type of Aggregate, quality of water, mix proportion, method of mixing, placing, compacting, temperature and curing method, & its duration.

## Cement

- Hydraulic cement - more commonly known as cement

It is also referred to as Portland or OPC - basic civil engg. material

- constituents { argillaceous material - clay  
 calcareous material - main ingredient  
 calcium carbonate

OPC { 33 Grade  
 43 Grade  
 53 Grade

28 day comp. strength is not less than 33 N/mm<sup>2</sup>, 43 N/mm<sup>2</sup> & 53 N/mm<sup>2</sup>.

## Physical properties of OPC

Grade of cement	Fineness m <sup>2</sup> /kg	Soundness mm	Setting time		28 day comp. strength N/mm <sup>2</sup>
			Initial (min)	Final (min)	
33	225	10	30	600	33
43	225	10	30	600	43
53	225	10	30	600	53

Rapid hardening  
 Portland pozzolona  
 Low heat cement

sulphate resistant cement  
 hydrophobic cement

Aggregate → 70 to 80% in volume of concrete

Concrete phase material   
 ↙ Paste phase (connected with cement)   
 ↘ Aggregate phase (aggregate)

classification   
 Aggregate   
 ↙ Normal weight Aggregate   
 ↘ Light weight Aggregate (Blast furnace slag)   
 ↘ Heavy weight Aggregate   
 ↙ natural   
 ↘ artificial   
 ↙ fine   
 ↘ coarse

Natural Aggregate → sand, gravel & crushed rock (such as granite, quartzite, basalt, sand stone)   
 Artificial Aggregate → Broken brick, Air cooled slag, sintered fly ash, bloated clay

### Coarse Aggregate

- Produced by disintegration of rock & crushing of rock
- retained on IS 4.75 mm sieve
- hard & durable (granite, basalt, quartzite)   
 Good C.A
- Light weight aggregate (Blast furnace slag)
- Brick-bats used for lime concrete or temporary or cheap concrete work
- For RCC - crushed rock aggregates used.
- Fineness modulus of C.A b/w 6 to 8.5   
 (Good concrete value)
- Aggregates - absolutely clean, free from organic matter & other impurities.



## Fine Aggregate

- obtained from rivers or lakes (some times, beach sand used)
- some places - sand is not available }  
large quantity sand required } - crushed stone dust used.
- Fineness modulus of sand - 2 to 3.2
- F.A assist cement paste - it helps - increases plasticity of concrete & prevent segregation of C.A
- Fine sand - 0.425 mm to 0.075 mm) - provide more cohesion & less sand is needed.
- Very fine & Very coarse sand not recommended for structural concrete

## Water

- used for mixing & curing of cement concrete.
- free from harmful substances (oil, acid, alkalies, salt, sugar, organic materials)
- Any portable water is <sup>generally</sup> satisfactory for both mixing & curing work.
- pH is b/w 6 & 8 - acceptable & suitable

Permissible limits for solids. IS: 456:2000 - P.No: 15  
Table: 1

Solid type	Permissible limit max. (mg/lit)
Organic	200
In-organic	3000
Sulphates ( $SO_3$ )	400
Chlorides Cl	2000 (with no embedded steel)
"	500 (RCC work)
Suspended matter	2000

# Grades of concrete

IS: 456: 2000 - P. NO: 16; Table: 2

Group	Grade	specified characteristic comp. strength of 150 mm cube @ 28 days N/mm <sup>2</sup>	Ratio
Ordinary concrete	M10	10	
	M15	15	
	M20	20	
Normal/ Standard concrete	M25	25	
	M30	30	
	M35	35	
	M40	40	
	M45	45	
	M50	50	
	M55	55	
High Strength concrete	M60	60	
	M65	65	
	M70	70	
	M75	75	
	M80	80	

M → Mix

→ Number specified comp. strength of 150 mm <sup>size</sup> cube @ 28 days

Minimum grade of RCC → M20

## Mix Ratio

M5 → 1:5:10

M7.5 → 1:4:8

M10 → 1:3:6

M15 → 1:2:4

M20 → 1:1.5:3

M25 → 1:1:2

Mass concrete work

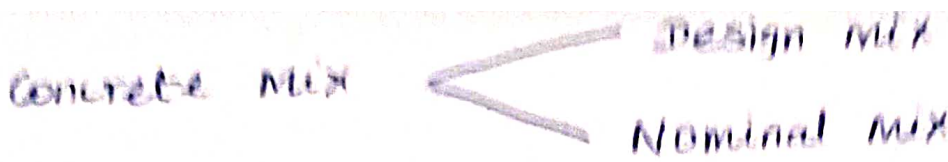
## Clean cover

Footing - 50 mm

Column - 40 mm

Beam - 25 mm

Slab - 15 mm



Nominal mix proportion

Grade of concrete

M20

Types of cement concrete

Grade	Max quantity of dry aggregate per sack of cement	F.A to C.A Ratio by mass	Max amt of water in lit
M5	400	generally	60
M7.5	625	1:2	45
M10	480	but may	34
M15	330	Varies from	32
M20	250	1:1.5 to 1:2.5	30

Plain cement concrete → Compression  
→ simple foundation

RCC → steel reinforcement taking tension, bending, torsion, fatigue

Mild steel bar, medium tensile <sup>steel</sup> bar, Rolled deformed bar

Cold twisted bar, Hand drawn steel wire fabric or Rolled steel

uses → Slab, column, beam, footing, staircase, water tank, retaining wall, shell, dam etc.

Prestressed concrete → High tensile wires are used as reinforcement instead of MS bar

→ Stretched initially to desired level then concrete placed.

uses → railway sleeper, electric pole etc. Large span bridge, beam etc.

Light weight concrete → less density



Pumice, expanded slates  
furnace clinker.

→ omitting F.A (no fine concrete) <sup>low drying shrinkage</sup>

→ using Light weight aggregate

→ by aerating concrete

(Gas, porous or foamed concrete)

**Precast cement concrete** → casting of various structural components with or without reinforcement

- Fresh concrete placed & compacted in mould for different structural units.
- After setting - curing
- uses — shelves, electrical or fencing poles, lintels, Jallies, railing, staircase steps, beam, sunshade etc.

**Fibre reinforced concrete** → combining cement fibre, sand & water.

- Asbestos, glass, nylon or coconut fibre - used as alternative to steel

**Advantages**

- (i) thin section can be formed, (ii) high durability,
- (iii) less cost of production (iv) less cost of maintenance.

**High density concrete** →

**Polymer concrete** → composite material of monomers, large sized aggregates, epoxy resins, methyl methacrylate, sand, ceramic powder etc

- organic content <sup>clear</sup> 10%
- polymer concrete attains 80% of strength within 1 day compared to ordinary concrete.

It is Resist high tension,  
lighter & less prone <sup>to</sup> crack  
highly resistant to chemical attack

# Properties of concrete

Weight → PCC —  $24 \text{ kN/m}^2$   
RCC —  $25 \text{ kN/m}^2$

Compr. Strength → min age of member when full design load/stress is expected (months)      Age factor

w/c ratio  
IS: 456:2000 —  
table: 2

1	1
3	1.1
6	1.15
12	1.20

Increase in strength with age → depends on grade & type of cement curing & environmental conditions

Tensile strength of concrete → flexural + splitting tensile strength  
IS: 456:2000 P.No: 16, clause: 6.2.2

flexural strength  $f_c = 0.7 \sqrt{f_{ck}} \text{ N/mm}^2$

Elastic deformation  
P.No: 16; clause: 6.2.3

$$E_c = 5000 \sqrt{f_{ck}}$$

(Actual measured value may be differ by 20%)

Workability of concrete — P.No: 17 (IS: 456-2000)

Loads & forces — IS: 456:2000 — P.No: 32

Shrinkage of concrete → IS: 456-2000; cl: 6.2.4.1, P.No: 16

time dependent deformation — generally compressive in nature

Factors influencing → constituents of concrete, size of the member, environmental conditions.

Total shrinkage of strain taken for design 0.0003

Creep of concrete → Factors influencing → properties of concrete, w/c ratio, Humidity & temperature of curing, Age of concrete at 1<sup>st</sup> loading, surface volume ratio

Slab

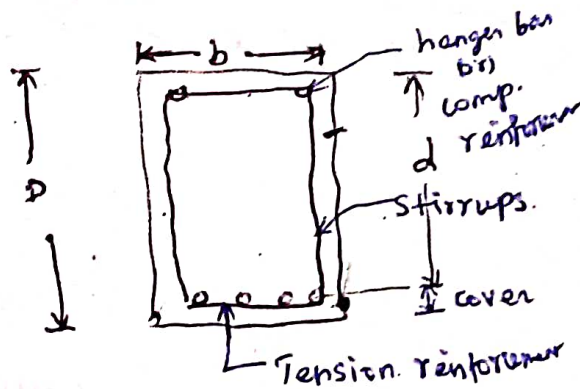
- 1) Minimum reinforcement  $\neq 0.15\% A_g$   
 P.N.O: 48  
 Cl: 26.5.2.1  
 $\neq 0.12\%$  c/s Area for HYSD bar.
- 2) Spacing  $\neq 3d$  or 300 mm (which ever is smaller for reinforcement)  
 P.N.O: 46  
 Cl: 26.3.3  
 $\neq 5d$  or 450 mm (which ever is smaller for distribution reinforcement)
- 3) Nominal cover  $\rightarrow \neq 15$  mm nor  $<$  dia of bar
- 4) Max. diameter of reinforcing bar  $\neq \frac{1}{8}$  depth of slab

Beam

P.N.O: 46

- 1) Tension reinforcement  
 min. area of tension reinforcement  $\frac{A_{st}}{bd} = \frac{0.85}{f_y}$   
 comp. reinforcement shall not exceed  $0.04 bd$   
 P.N.O: 47
- Min. Tension reinforcement  $\neq 0.34\%$  for MS ( $f_y = 250$ )  
 $\neq 0.20\%$  for HYSD ( $f_y \geq 15$ )

Max. reinforcement  $\rightarrow \neq 0.04 bd$   
(both tension & compression)



Spacing b/w bars

- (i)  $\neq$  dia of larger bar
- (ii)  $\neq$  less than normal max. size of coarse aggregate + 5 mm

Take greater value.

Nominal cover  $\rightarrow \neq 25 \text{ mm}$   
less than dia of bar

Side face reinforcement  $\rightarrow$  P.No: 47

- \* Overall depth of web in a beam exceeds 750 mm — side reinforcement required
- \* 0.1% of web area should be distributed equally on the two faces at a spacing not exceeding 300 mm or web thickness (whichever is less)

Shear reinforcement

P.No. 48 min

$$\frac{A_{sv}}{bS} = \frac{0.4}{0.87 f_y}$$

Labels:   
- stirrups (pointing to  $A_{sv}$ )  
- breadth of beam or web (pointing to  $b$ )  
- Spacing of stirrups (pointing to  $S$ )

Column

P.No. 48  
Cl: 26-5.3.1

min reinforcement  $\neq 0.8\%$  c/s area of column  
max. reinforcement  $\neq 6\%$

Minimum no. of bar  $\left\{ \begin{array}{l} \text{Rectangular column (4)} \\ \text{Circular (5)} \end{array} \right.$

Dia of bar  $\neq 12 \text{ mm}$

min cover  $\rightarrow 40 \text{ mm}$   
dia of bar } greater provided

Lateral ties

Diameter  $\rightarrow \neq \frac{1}{4} \phi$  of longest longitudinal bar } greater value  
5 mm

Pitch

- Least lateral dimension of the member
- 16 times the smallest diameter of longitudinal bar
- 300 mm.

Design methods

- working stress
- Ultimate Load method
- Limit state method.

Working stress (or) Elastic stress method or Modulus ratio method.

\* based on elastic theory

\* both concrete & steel ~~are~~ <sup>considered</sup> — behave in linear elastic manner in all states.

\* <sup>Adopt.</sup> Permissible stress or working stress in each material

\* applying certain specific F.O.S. on strength <sup>of material</sup> for design purposes.

\*  $F.O.S = 3$  cube strength of concrete

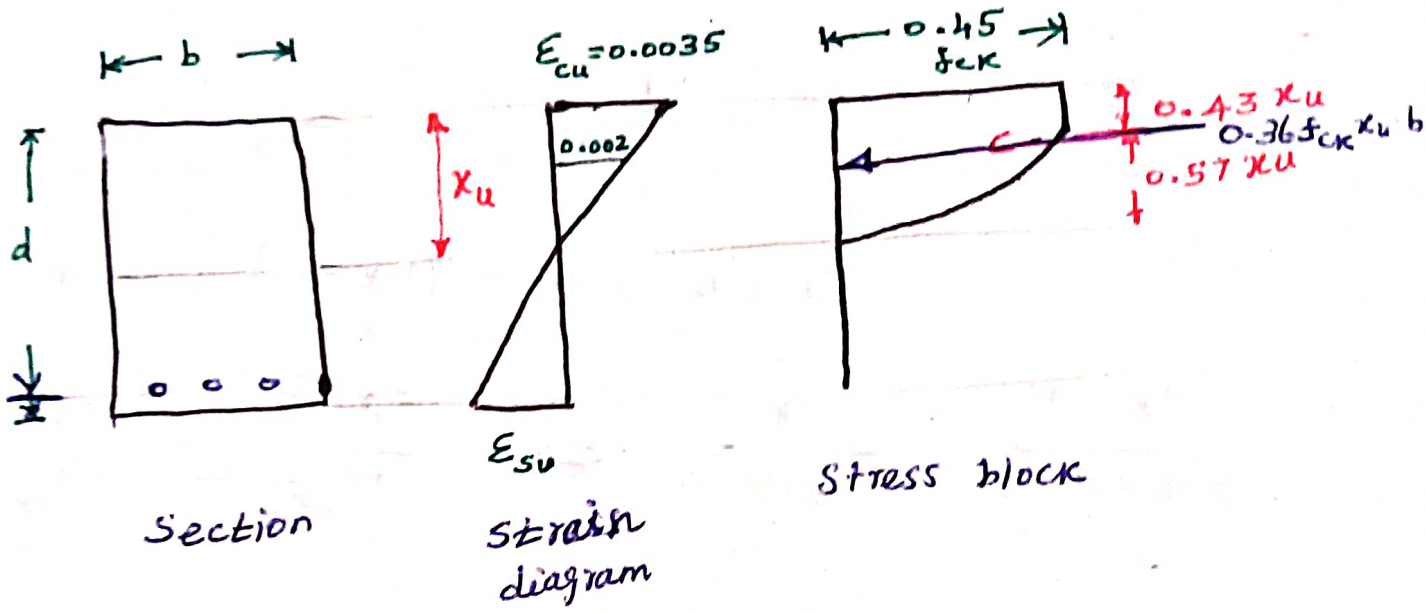
$F.O.S = 1.8$  yield strength of steel

Assumptions p. NO: 80

\* at any c/s, plane sections before bending re



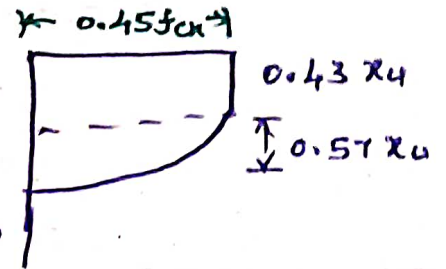
# Equivalent compression block in concrete



Area of stress block

= Area of  $\square^{lm}$  portion + Area of Parabolic portion

=  $(0.45 f_{ck} \times 0.43 f_{ck}) + \frac{2}{3} (0.45 f_{ck} \times 0.57 f_{ck})$



Area of Stress block =  $0.36 f_{ck} \cdot x_u$

in steel

Resultant comp. stress = Area of comp. stress  $\times$  width of  $\square^{lm}$  beam

$C = 0.36 f_{ck} \cdot x_u \cdot b$

To find Resultant Tensile force (T)

$T = \text{Area of steel} \times \text{stress in steel}$   
 $= A_{st} \cdot 0.87 f_y$

$T = 0.87 f_y \cdot A_{st}$

## Depth of neutral Axis

$$C = T$$

$$0.36 f_{ck} \cdot x_u \cdot b = 0.87 f_y \cdot A_{st}$$

$$x_u = \frac{0.87 f_y \cdot A_{st}}{0.36 f_{ck} \cdot b}$$

Note  
 $\frac{f_y}{1.15} = 0.87 f_y$

## Limiting value of $x_u$

$$\frac{\epsilon_{cu}}{x_u} = \frac{\epsilon_{su}}{d - x_u}$$

$$\epsilon_{cu} = 0.0035$$

max. strain in

Tension  $\epsilon_{su} = \frac{f_y}{1.15 E_s} + 0.002$

$$E_s = 2 \times 10^5 \text{ N/mm}^2$$

\*) Assume,  $F_e 250$ ; M15 concrete

$$f_y = 250; f_{ck} = 15$$

$$\epsilon_{su} = \frac{250}{1.15 \times 2 \times 10^5} + 0.002 = 0.00308$$

$$\frac{\epsilon_{cu}}{x_u} = \frac{\epsilon_{su}}{d - x_u} \Rightarrow \frac{0.0035}{x_u} = \frac{0.00308}{d - x_u}$$

$$0.0035(d - x_u) = 0.00308 x_u$$

$$\frac{x_u}{d} = 0.53$$

\*\* Fe 415 & M20

$$\epsilon_{su} = \frac{415}{1.15 \times 2 \times 10^5} + 0.002 = 0.0038$$

$$\frac{0.0035}{x_u} = \frac{0.0038}{d - x_u}$$

$$\frac{x_u}{d} = 0.48$$

Max. % of Reinforcement ( $P_t$ )

Assume the section is Balanced

$$\therefore \frac{x_u}{d} = \frac{x_{u \max}}{d}$$

$$\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} \cdot b \cdot d} \quad \text{--- (1)}$$

Assume Fe 250 & M15

$$\frac{x_{u \max}}{d} = 0.53 \quad \text{--- (2)}$$

Equating (1) & (2)

$$\frac{0.87 f_y A_{st}}{0.36 f_{ck} b d} = 0.53$$

$$A_{st} = \frac{0.53 \times 0.36 f_{ck} b d}{0.87 f_y}$$

$$\frac{A_{st}}{bd} = \frac{0.53 \times 0.36 \times 15}{0.87 \times 250}$$

$$\frac{A_{st}}{bd} = 0.013$$

$$\% \text{ of steel } (P_{t \min}) = 100 \frac{A_{st}}{bd} = 100 \times 0.013$$

$$\% \text{ of steel } P_{t \min} = 1.3\%$$

$$f_y = 250 ; f_{ck} = 15$$

If M<sub>20</sub> & Fe 415

$$\frac{x_{u \max}}{d} = 0.48 \quad \text{--- (1)}$$

$$\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} bd} \quad \text{--- (2)}$$

$$\text{(1)} = \text{(2)}$$

$$0.48 = \frac{0.87 f_y A_{st}}{0.36 f_{ck} bd}$$

$$\frac{A_{st}}{bd} = 0.00957$$

$$\% \text{ of steel } (P_{t \min}) = 100 \frac{A_{st}}{bd}$$

$$P_{t \min} = 0.957\%$$

$$M_{20} \& Fe 415$$

# Working stress method (or) Allowable (or) Permissible stress method

Elastic design method

\* Members are designed to never go beyond their elastic range.

<p><u>Working stress method (Elastic)</u></p>	<p><u>Limit state Method (Plastic)</u></p>
<p>1. Based on <u>elastic theory</u>                      * concrete &amp; steel are assumed to <u>act together elastically</u> and follow <u>Hook's law</u> (linear stress-strain relationship)</p>	<p>It takes <u>into account</u> actual <u>non-linear elasto-plastic stress-strain behaviour</u> of <u>concrete and steel</u></p>
<p>2. The stresses in the structural members are considered for <u>normal working loads</u> <u>without considering the conditions existing at the time of failure.</u></p>	<p>The design is based on <u>Ultimate loads at failure.</u>  <math display="block">\text{Ultimate load} = \text{working load} \times \text{Partial Safety Factor}</math>                     Note: → Different partial safety factors are used for limit state of collapse &amp; limit state of serviceability under different load combinations.</p>
<p>3. <u>Design criteria</u></p> <p>(i) Load carrying capacity <sup>greater than</sup> <math>&gt;</math> working load</p> <p>(ii) Stresses in material (concrete &amp; steel) <sup>less than</sup> <math>&lt;</math> permissible stresses in material</p> <p>Permissible stress = <math>\frac{\text{characteristic cube strength of concrete}}{\text{FOS of concrete}}</math> (or) <math>\frac{\text{Yield strength of steel}}{\text{FOS of steel}}</math></p> <p>i.e., <math>\frac{\text{Cube strength (or) } f_y}{\text{FOS of concrete (or) steel}}</math></p>	<p><u>Design criteria</u></p> <p><math>\frac{\text{Load carrying capacity}}{\text{Partial safety factor for material}} &gt;</math> Load combination comprising of <math>\leq</math> (Load <math>\times</math> corresponding partial safety factor for the level)</p>
<p>4. It follows a <u>deterministic approach.</u>                      It assumes that the loads, FOS &amp; permissible stresses are accurately known.</p>	<p>It follows a <u>non-deterministic approach.</u>                      (It adopts probable load &amp; probable strength of materials as per actual or based on experience or observations depending upon the situation.)</p>
<p>5. Material strength are not fully utilized in designing the member.</p>	<p>Material strength are fully utilized in designing the member.</p>
<p>The stresses are obtained from design loads &amp; compared with design strength.                      The ultimate stresses of material itself are used as allowable stresses. The capabilities are not under estimated</p>	<p>as much as they are in working stress. Partial Safety Factors are used in LEM</p>

## Slab

longer span  
shorter span  
 $\frac{l_y}{l_x} \geq 2$  one way slab

$\frac{l_y}{l_x} < 2$  Two Way slab

Span  
depth

ratio

Cantilever — 7  
SS — 20  
Continuous — 26

} Also applicable for beam

\* % of reinforcement — low in the range of 0.3 to 0.5%

\* Hence Modification factor ( $K_t$ ) — tension reinforcement result  
in span/depth ratio (25 to 30 for one way slab)

Design Methods <sup>Concrete structural members designed by</sup> → Theoretical or experimental investigations.  
\* Mostly used theoretical method.

1. Working stress Method
2. Ultimate load method
3. Limit state method.

Working stress method: (or) Elastic stress method  
(or) Modular ratio method.

- \* based on elastic theory.
- \* concrete & steel — behave in linear elastic manner
- <sup>WSM</sup> \* Adopt permissible stress (or) working stress in each material & applying specific F.O.S on the strength of material for design purpose.
- \* F.O.S  $\frac{f_c}{3}$  → strength of concrete (cube strength)
- \* F.O.S = 1.8 → yield strength of steel

### Assumptions

- \* <sup>or</sup> any c/s, plane sections before bending remain plane after bending.
- \* All tensile stresses are taken up by reinforcement & none by concrete.
- \* The stress-strain relationship of steel & concrete, under working load is a straight line
- \* Modular ratio  $m = \frac{280}{3\sigma_{bc}}$  due to bending in concrete  
↑ permissible comp. stress

## Limitations

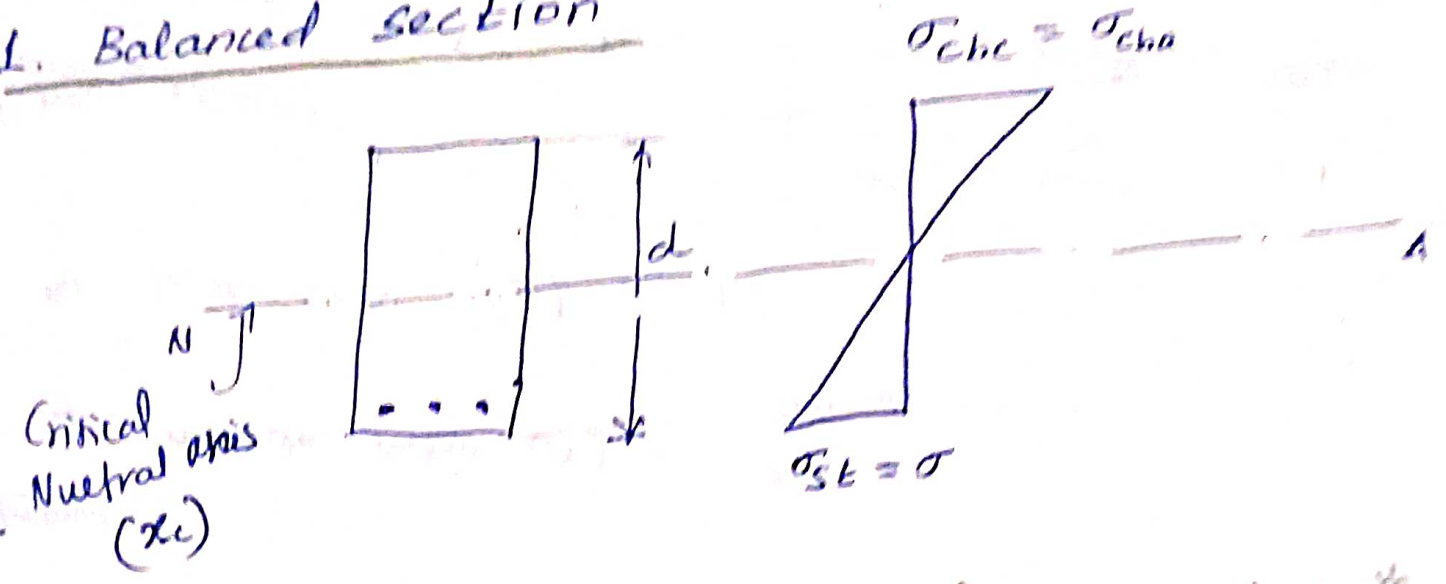
- \* <sup>NSM</sup> only elastic behaviour (real strength not provides, the correct FOS of the structural member against failure)
- \* Modular ratio — design approach results in longer % of compression steel.
- \* constant modular ratio is not true as the concrete has a non-linear stress strain behaviour
- \* Concrete members have variable moment of Inertia due to variable cracking along their length & varying area of reinforcement
- \* This method does not give correct picture of the safety of the members.



Modular ratio  $m = \frac{E_{st}}{3\sigma_{cbc}}$  ← max. permissible stress in concrete

Types of Reinforced concrete sections ( $x_a = x_c$ )

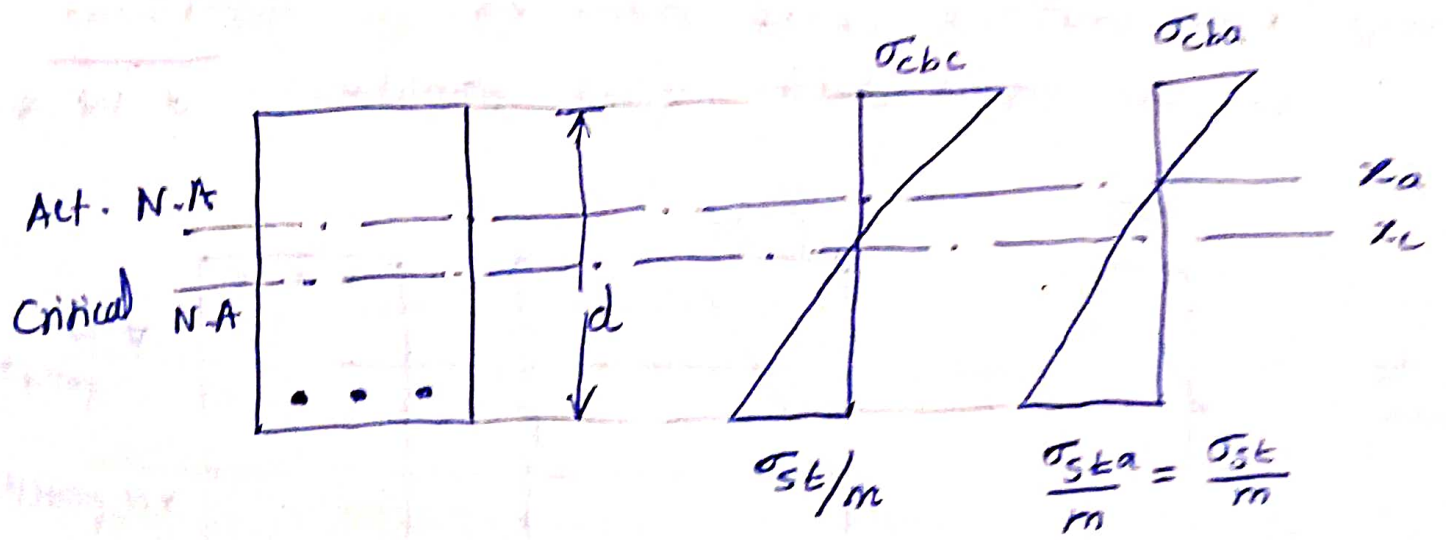
1. Balanced section



2. Under reinforced section

$x_a < x_c$

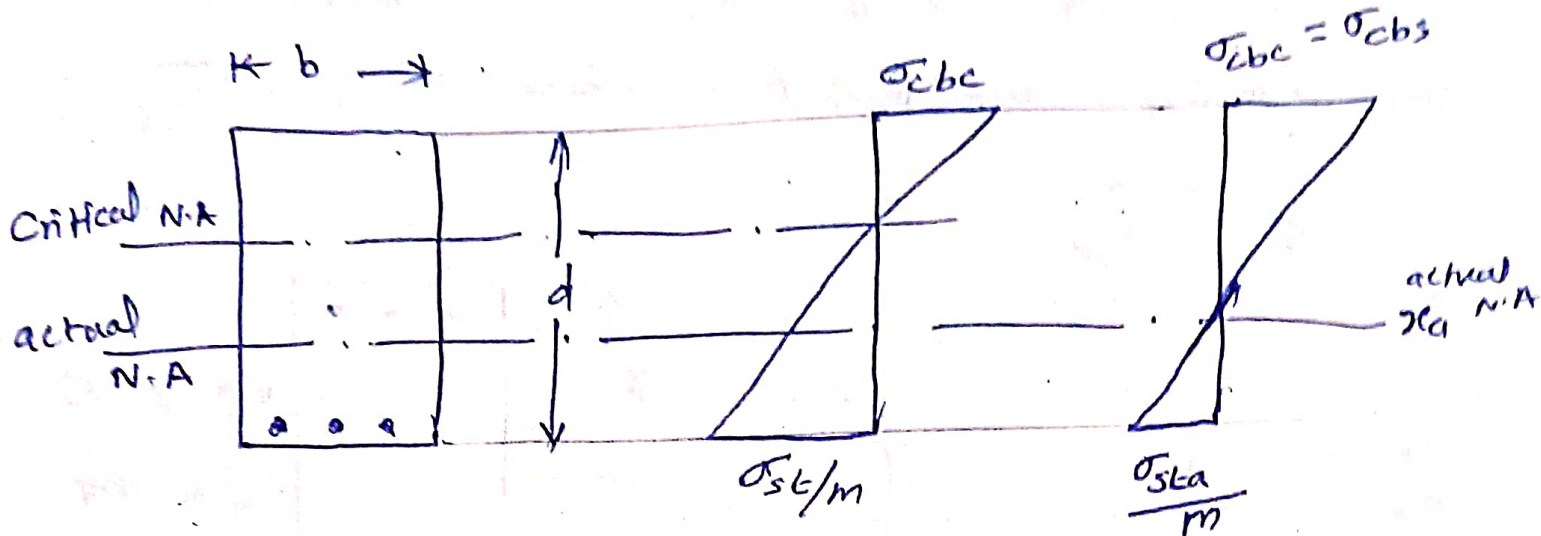
or N.A  
 ↓  
 $x_a < x_c$   
 CRITICAL N.A. depth



\* Moment of resistance computed from tension side with steel reaches max. permissible stress  $\sigma_{st}$ .

### 3. over reinforced section

$$x_a > x_c$$



\* M.R. computed from comp-side as the concrete at extreme fibre reaches max. Permissible Stress  $\sigma_{cbc}$ .

$$M.R = \frac{\sigma_{bc}}{2} \cdot x_a \cdot b \cdot \left( d - \frac{x_a}{3} \right)$$

#### Formula

comp. stress in concrete

1) modular ratio (m) =  $\frac{280}{3\sigma_{cbc}}$

2) Lever arm (j) = 1 -

#### $\sigma_{cbc}$ values

Concrete grade	$\sigma_{cbc}$
M10	3
M15	5
M20	7
M25	8.5
M30	10
M35	11.5
M40	13

#### $\sigma_{st}$ values

Steel grade	$\sigma_{st}$
MS/Fe 250	140
Fe 415	230

# Analysis & Design of singly Reinforced Rectangular beam

by WSM

- 1) A reinforced concrete beam of rectangular section 300 mm by 650 mm overall depth is reinforced with 4 bars of 32 mm diameter at an effective depth of 600 mm. Using M20 grade concrete & Fe415 HYSD Bars. Estimate the moment of resistance of the section.

Step: 1 - Given Data:

$$\begin{aligned}b &= 300 \text{ mm} \\d &= 600 \text{ mm} \\D &= 650 \text{ mm} \\f_{ck} &= 20 \text{ N/mm}^2 \\f_y &= 415 \text{ N/mm}^2\end{aligned}$$

Step: 2 - To find

$$M_R = ?$$

Step: 3 -  $A_{st}$

$$A_{st} = 4 \times \frac{\pi \times 32^2}{4} = 3216 \text{ mm}^2$$

Step: 4 - Permissible stresses

$$\begin{aligned}\sigma_{cbc} &= 7 \text{ N/mm}^2 \\ \sigma_{st} &= 230 \text{ N/mm}^2\end{aligned}$$

Step: 5 - Modular ratio (m)

$$m = \frac{280}{3\sigma_{cbc}} = \frac{280}{3 \times 7} = 13.33$$

Step: 6 - Actual neutral axis depth ( $x_a$ )

$$b \cdot \frac{x_a^2}{2} = m \cdot A_{st} (d - x_a)$$

$$300 \times \frac{x_a^2}{2} = 13.33 \times 3216 \times (600 - x_a)$$

$$x_a = 295.16 \text{ mm}$$

## Unit - I - Working stress Method

IS 456:2000 - P. No. 81, Table 21 <sup>due to bending</sup>

\*  $\sigma_{cbc}$  → Permissible stresses in concrete (N/mm<sup>2</sup>)

Grade of concrete	M10	M15	M20	M25	M30	M35	M40	M45	M50
$\sigma_{cbc}$ (bending)	3	5	7	8.5	10	11.5	13	14.5	16

IS 456:2000 - P. No. 82

$$m \sigma_{cbc} = \frac{280}{3 \sigma_{cbc}}$$

$m$  → modular ratio  
 $\sigma_{cbc} = \frac{1}{3} f_{ck}$

\*  $\sigma_{st}$  → Permissible <sup>compressive</sup> stresses for steel in tension

IS: 456:2000  
 P. No: 82 ; Table: 22

\* Actual neutral axis ( $x_a$ )

$$b \cdot \frac{x_a^2}{2} = m \cdot A_{st} \cdot (d - x_a)$$

\* Critical neutral axis ( $x_c$ )

$$x_c = \left[ \frac{1}{1 + \frac{\sigma_{st}}{m \cdot \sigma_{cbc}}} \right] d \quad (\text{or})$$

$$x_c = \left[ \frac{m \cdot \sigma_{cbc} \cdot d}{m \cdot \sigma_{cbc} + \sigma_{st}} \right]$$

Singly Reinforced section

\* If  $x_a > x_c$  over reinforced section

Comp force

$$M_R = \frac{\sigma_{cbc}}{2} \cdot b \cdot x_a \cdot \left( d - \frac{x_a}{3} \right)$$

\* If  $x_a < x_c$  under reinforced section

Tensile force

$$M_R = \sigma_{st} \cdot A_{st} \cdot \left( d - \frac{x_a}{3} \right)$$

\* If  $x_a = x_c$  , balanced section

Problem 1

1) A simply supported reinforced concrete one way slab with an effective span of 4 m is reinforced with 10mm diameter Fe 415 HYSD bars spaced at 200mm centres at an effective depth of 180mm. Using M20 grade concrete, Calculate the maximum permissible live load on the slab if the self weight of slab and finishes are 5.5 kN/m<sup>2</sup>.

Given Data

- Effective span (L) = 4 m
- Effective depth (d) = 180 mm
- Spacing of 10mm diameter bars = 200 mm
- Materials: M20 grade concrete  
Fe 415 HYSD bars

1000 mm / 200 spacing = 5 Nos.

$$A_{st} = 5 \times \frac{\pi d^2}{4}$$

$$A_{st} = 395 \text{ mm}^2/\text{m}$$

\*  $\sigma_{cbc}$  values for M20

$$\sigma_{cbc} = 7 \text{ N/mm}^2$$

$\sigma_{st}$  values for Fe 415

$$\sigma_{st} = 230 \text{ N/mm}^2$$

\* modular ratio (m)  $m = \frac{280}{3 \sigma_{cbc}} = 13$

\* critical neutral axis ( $x_c$ )  $x_c = \frac{m \cdot \sigma_{cbc} \cdot d}{m \cdot \sigma_{cbc} + \sigma_{st}} = 0.284 d$

$$x_c = 51.12 \text{ mm}$$

\* Loads

$$M_g = \frac{W_g \cdot l^2}{8} = \frac{5.5 \times 4^2}{8} = 11 \text{ kN/m}$$

\* ( $x_a$ ) Neutral axis depth

$$b \cdot \frac{x_a^2}{2} = m \cdot A_{st} \cdot (d - x_a)$$

b = 1000 mm  
m = 13  
A<sub>st</sub> = 395 mm<sup>2</sup>/m  
d = 180 mm

$$x_a = 38.2 \text{ mm}$$

\*  $x_c > x_a$ , under reinforced section

\*  $M_R = \sigma_{st} \cdot A_{st} \left( d - \frac{x_a}{3} \right) = 15.2 \times 10^6 \text{ N.mm}$

\* Live load moment

$$M_l = (M_R - M_g) = 15.2 - 11 = 4.2 \text{ kN.m}$$

$$M_l = \frac{W_l \cdot l^2}{8}$$

$$W_l = 2.1 \text{ kN/m (permissible live load on slab)}$$

- 2) A simply supported R.C. slab having an overall thickness 150mm is reinforced with 12mm diameter bars at an effective depth of 130mm. The spacing of the bars is 100mm. The effective span of the slab = 4m. If the self weight of slab and finishes is  $4.2 \text{ kN/m}^2$ , estimate the maximum permissible live load on the slab. Adopt M15 grade concrete and Fe 250 grade I-Steel.

Data.

$$L = 4\text{m} ; \quad D = 150\text{mm} ; \quad d = 130\text{mm} ;$$

spacing of 12mm diameter bars = 100mm

$$A_{st} = 10 \times \frac{\pi \times 12^2}{4} = 1130.97 \text{ mm}^2$$

Materials: M15 grade concrete & Fe 250 grade I. Mild steel

Permissible stresses,  $\sigma_{cbc}$  for M15 ;  $\sigma_{cbc} = 5 \text{ N/mm}^2$

$\sigma_{st}$  for Fe 250 ;  $\sigma_{st} = 140 \text{ N/mm}^2$

$$\text{modular ratio } m = \frac{280}{3 \sigma_{cbc}} = 19$$

$$\underline{x_c} = \frac{m \cdot \sigma_{cbc} \cdot d}{m \cdot \sigma_{cbc} + \sigma_{st}} = 0.4 \times 130 = \underline{52 \text{ mm}}$$

$$\text{Dead load moment, } \underline{M_g} = \frac{W_g l^2}{8} = \frac{4.2 \times 4^2}{8} = \underline{8.4 \text{ kN.m}}$$

$x_a$

$$b \cdot \frac{x_a^2}{2} = m \cdot A_{st} (d - x_a)$$

$$b = 1000\text{mm}$$

$$m = 19$$

$$d = 130\text{mm}$$

$$A_{st} = 1130.97 \text{ mm}^2$$

$$\underline{x_a = 56 \text{ mm}}$$

$x_a > x_c$ , over reinforced section

$$M_R = \frac{\sigma_{cbc}}{2} \cdot x_a \cdot b \cdot \left(d - \frac{x_a}{3}\right) = 15.58 \text{ kN.m}$$

$$\text{Live load moment } \underline{M_q} = M_R - M_g = \underline{7.18 \text{ kN.m}}$$

$$\text{Live load } \cdot W_q = 3.59 \text{ kN/m}^2$$

3) A reinforced concrete beam having a rectangular section 300mm wide is reinforced with 2 bars of 12mm diameter at an effective depth of 550mm. The section is subjected to a service load moment of 40 kN.m. Assuming M20 grade concrete and Fe 415 HYSD bars, estimate the stresses in concrete and steel.

Given data

$b = 300 \text{ mm}$  ;  
 $d = 550 \text{ mm}$  ;  
 Service load moment  $M = 40 \text{ kN.m}$

Materials

M20 grade concrete  
 $\sigma_{cbc} = 7 \text{ N/mm}^2$   
 Fe 415 HYSD bars  
 $\sigma_{st} = 230 \text{ N/mm}^2$

\*  $m = \frac{280}{3 \sigma_{cbc}} = 13$

\*  $\frac{x_a}{2} \cdot b \cdot \frac{x_a^2}{2} = m \cdot A_{st} \cdot (d - x_a)$   
 $x_a = 94.5 \text{ mm}$

\*  $x_c = \frac{m \cdot \sigma_{cbc} \cdot d}{m \cdot \sigma_{cbc} + \sigma_{st}}$   
 $x_c = 156.2 \text{ mm}$

\*  $x_a < x_c$ , under reinforced section

\* stresses in concrete & steel.

Taking moment about the tension steel centroid

Concrete  $M_R = \frac{\sigma_{cbc}}{2} \cdot b \cdot x_a \cdot \left( d - \frac{x_a}{3} \right)$   
 $\sigma_{cbc} = 5.44 \text{ N/mm}^2$

Taking moment about line of action of comp. force in concrete

Steel  $M_R = \sigma_{st} \cdot A_{st} \cdot \left( d - \frac{x_a}{3} \right)$   
 $\sigma_{st} = 340.8 \text{ N/mm}^2$

4) A reinforced concrete beam of span 5m has a rectangular section 250mm wide by 500mm depth. The beam is reinforced with 3 bars of 16mm diameter on the tension side at an effective depth of 450mm & 2 bars of 16mm diameter on the compression side at a cover of 50mm from the compression face. If M15 grade concrete & Fe 250 grade mild steel is used, estimate the maximum permissible live load on the beam.

### Given data

$$\begin{aligned}L &= 5 \text{ m} \\ b &= 250 \text{ mm} \\ D &= 500 \text{ mm} \\ d &= 450 \text{ mm} \\ d_c &= 50 \text{ mm}\end{aligned}$$

$$\begin{aligned}A_{st} &= 3 \times \pi \times \frac{16^2}{4} = 603 \text{ mm}^2 \\ A_{sc} &= 2 \times \pi \times \frac{16^2}{4} = 402 \text{ mm}^2\end{aligned}$$

### Materials

M15 grade concrete  $\sigma_{cbc} = 5 \text{ N/mm}^2$   
Fe 250 grade mild steel,  $\sigma_{st} = 140 \text{ N/mm}^2$

$$* m = \frac{280}{3 \sigma_{cbc}} = 19$$

$$* x_c = \frac{m \cdot \sigma_{cbc} \cdot d}{m \cdot \sigma_{cbc} + \sigma_{st}} = 180 \text{ mm}$$

$$* \underline{x_a} \quad b \cdot \frac{x_a^2}{2} + (1.5m - 1) A_{sc} (x_a - d_c) = m \cdot A_{st} \cdot (d - x_a)$$

$$\underline{x_a = 142 \text{ mm}}$$

### \* Check

$x_c > x_a$ , under reinforced section

$$* M_R = \frac{\sigma_{cbc}'}{2} \cdot b \cdot x_a \left(d - \frac{x_a}{3}\right) + (1.5m - 1) A_{sc} \sigma_{cbc} (d - d_c)$$

$$\underline{M_R = 33.8 \text{ kN.m}}$$

$$\sigma_{cbc}' = \frac{\sigma_{st}}{m} \left(\frac{x_a}{d - x_a}\right)$$

$$\underline{\sigma_{cbc}' = 3.39 \text{ N/mm}^2}$$

$$\overline{\sigma_{cbc}} = \frac{\sigma_{st}}{m} \left(\frac{x_a - d_c}{d - x_a}\right)$$

$$\underline{\overline{\sigma_{cbc}} = 2.19 \text{ N/mm}^2}$$

### \* Permissible Live load

$$\text{Self wt of beam} = b \cdot D \cdot \gamma = 3 \text{ kN/m}$$

$$\text{Dead Load moment } M_g = \frac{W_g \cdot l^2}{8} = 9.375 \text{ kN.m}$$

$$M_q = M_R - M_g = 33.8 - 9.375$$

$$\underline{M_q = 24.4 \text{ kN.m}}$$

$$\text{Live Load, } \underline{W_q = 7.8 \text{ kN/m}}$$

$$\begin{aligned}M_q &= \frac{W_q \cdot l^2}{8} \\ 24.4 &= \frac{W_q \cdot 5^2}{8} \\ \therefore W_q &= 7.8 \text{ kN/m}\end{aligned}$$



1) Design the roof slab for a hall of size 4m x 10m by working stress method using M20 concrete and Fe415 steel. The slab is simply resting on 230mm thick brick wall assumed. Take  $1.1$  on the slab as  $1.5 \text{ kN/m}^2$  & finish load as  $2.25 \text{ kN/m}^2$ .

Step: 1 - Check for slab

$$\frac{l_y}{l_x} = \frac{\text{longer span}}{\text{shorter span}} = \frac{10}{4} = 2.5 > 2$$

Hence it is one way slab

The reinforcement will be provided along the shorter span. So the span will be taken as 4m.

Step: 2 - Design constants:-

$$x_c^{(00)} k = \frac{m \cdot \sigma_{cbc}}{m \cdot \sigma_{cbc} + \sigma_{st}} = 0.288$$

$$j = 1 - \frac{k}{3} = 0.904$$

$$Q = \frac{\sigma_{cbc}}{2} \cdot k \cdot j = 0.911$$

For M20 grade concrete  
 $\sigma_{cbc} = 7$ ;  $m = \frac{280}{3\sigma_{cbc}} = 13.33$   
 $\sigma_{st} = 230 \text{ N/mm}^2$

$$\alpha = \frac{\sigma_{cbc}}{2} \cdot x_c \cdot \left(1 - \frac{x_c}{3}\right)$$

Step: 3 - Dimensions of slab

$$\frac{l}{d} = B.V. \times K_f \cdot K_c$$

$$\frac{4000}{d} = 20 \times K_f \cdot K_c$$

$$\therefore d = 153.84 \text{ mm}$$

Use 10mm # bar

$$\therefore D = d + \frac{\phi}{2} + 15 = 173.84 \text{ mm}$$

$$D \approx 175 \text{ mm}$$

$$b = 1000 \text{ mm}$$

IS: 456-2000

Page No: 38; Fig: 4

$$f_x = 0.58 f_y = 0.58 \times 415 = 240.7$$

$K_f = 1.3$  → Assuming 0.4% of tensile reinforcement

$K_c = 1.00$  → From Fig. 5 - P.No: 39

Step: 4 - Effective span

$$(i) \text{ c/c of supports} = 4 + 0.23 = 4.23 \text{ m}$$

$$(ii) \text{ Clear span} + \text{Eff. depth} = 4 + 0.153 = 4.153 \text{ m}$$

} Take least value

$$\therefore l_{\text{eff}} = 4.15 \text{ m}$$

Step: 5 - self wt calculation

$$\text{self wt} = b D \gamma = 1 \times 0.175 \times 25 = 4.375 \text{ kN/m}$$

Step: 6 - Total load and Max B.M

$$W = LL + DL + FFL = (1.5 \times 1) + 4.375 + (2.25 \times 1)$$

$$W = 8.125 \text{ kN/m}$$

$$M_u = \frac{WL^2}{8} = \frac{8.125 \times 4.5^2}{8} = 17.49 \text{ kN.m}$$

Step: 7 - Max. SF

$$V = \frac{WL}{2} = \frac{8.125 \times 4.5}{2} = 16.85 \text{ k.N}$$

Step: 8 - check for eff. depth

$$d_{\text{provided}} = 153.84 \text{ mm}$$

$$d_{\text{req}} = \sqrt{\frac{M}{Q_b}} = \sqrt{\frac{17.49 \times 10^6}{0.911 \times 1000}} = 138.55 \text{ mm}$$

$d_{\text{provided}} > d_{\text{req}}$ , Hence the provided depth is adequate.

Step: 9 - Main reinforcement

$$A_{st} = \frac{M}{\sigma_{st} j d} = \frac{17.49 \times 10^6}{230 \times 0.904 \times 153.84} = 546.79 \text{ mm}^2$$

$$A_{st \text{ min}} = 0.12\% b D = \frac{0.12}{100} \times 1000 \times 175 = 210 \text{ mm}^2$$

$A_{st} > A_{st \text{ min}}$ , provided 10 mm  $\phi$  bar

$$\text{Spacing, } S = \frac{1000 a_{st}}{A_{st}} = \frac{1000 \times \pi \times 10^2 / 4}{546.79} = 143.63 \text{ mm} \approx 140 \text{ mm}$$

Hence provide 10 mm  $\phi$  bar @ 140 mm c/c.

Step: 10 - Distribution Reinforcement

$$A_{st} = 0.12\% b D = 210 \text{ mm}^2 \quad \text{Provide 8 mm } \# \text{ bar}$$

$$\text{Spacing } S = \frac{1000 a_{st}}{A_{st}} = \frac{1000 \times \pi \times 8^2 / 4}{210} = 239.36 \text{ mm}$$

Provide 8 mm  $\phi$  bars @ 230 mm c/c.

Step: 11 - check for shear

$$\tau_v = \frac{V}{bd} = \frac{16185 \times 10^3}{1000 \times 153.84} = 0.109 \text{ N/mm}^2$$

$\tau_c$

assume 50% of deviate reinforcement to be bent up near support

$$\rho = \frac{100 A_{sc}}{bd} = \frac{100 \times (546.79)}{1000 \times 153.84} = 0.355\%$$

$$0.15 \rightarrow 0.18$$

$$0.25 \rightarrow 0.22$$

$$\therefore 0.177 \rightarrow 0.18 + \left( \frac{0.22 - 0.18}{0.25 - 0.15} \right) (0.177 - 0.15)$$

$$\tau_c = 0.19 \text{ N/mm}^2$$

$$k \cdot \tau_c = 1.3 \times 0.19 = 0.247 \text{ N/mm}^2$$

$$\tau_{c \max} = 1.8 \times 0.15 = 0.9 \text{ N/mm}^2$$

$$\tau_v < \tau_c < \tau_{c \max} \quad \text{Hence safe.}$$

Step: 12 - check for deflection control.

$$\rho = \frac{100 A_{st}}{bd} = \frac{100 \times 546.79}{1000 \times 153.84} = 0.355\%$$

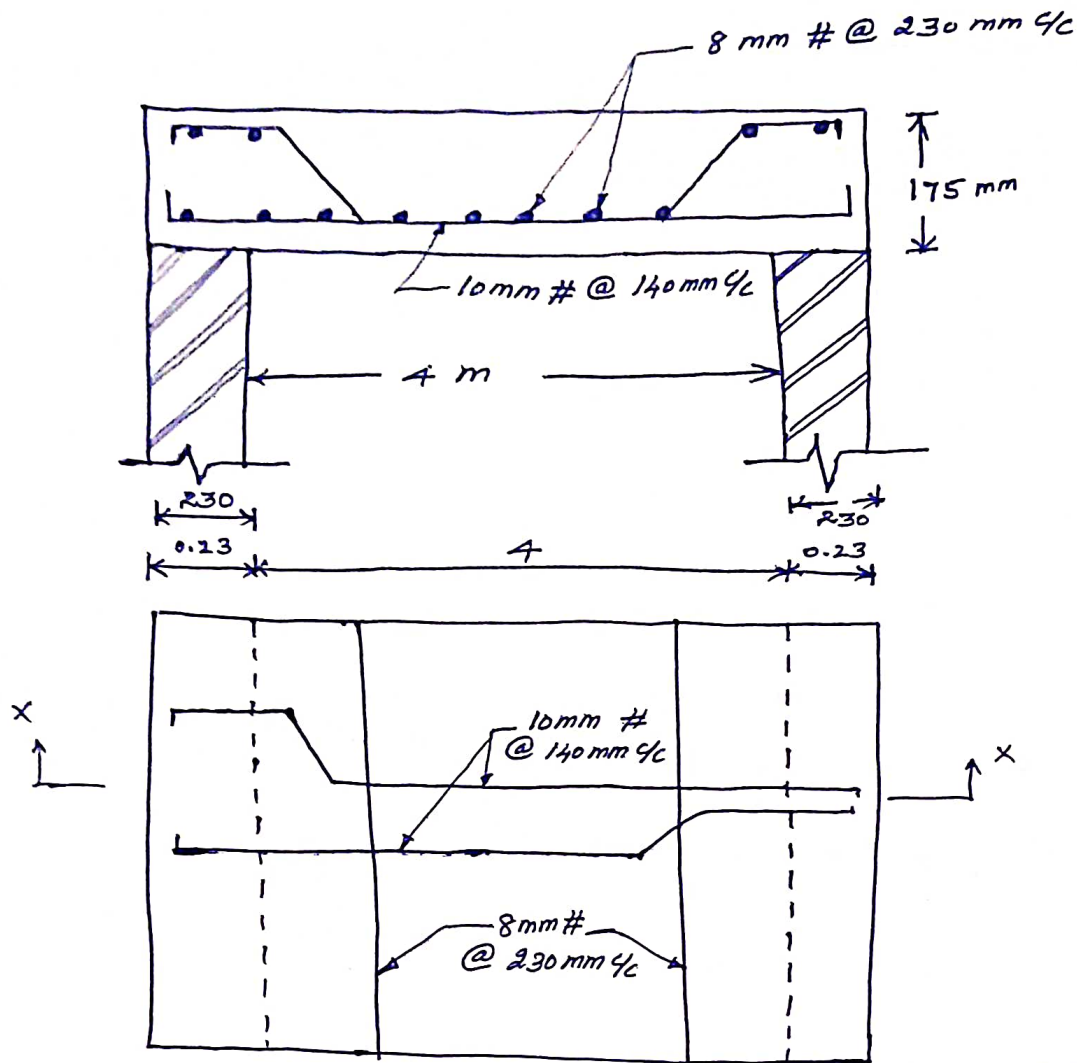
From IS: 456-2000 ; P.No: 38 ; fig: 4

$$k_L = 1.4$$

$$\left( \frac{L}{d} \right)_{\text{prov}} = \frac{4000}{153.84} = 26$$

$$\left( \frac{L}{d} \right)_{\text{max}} = 8V k_L \cdot k_c = 20 \times 1.4 \times 1 = 28$$

$$\left( \frac{L}{d} \right)_{\text{max}} > \left( \frac{L}{d} \right)_{\text{prov}} \quad \text{Hence safe.}$$



- 2) Design a two way slab for a residential floor to suit the following data.
- size of floor = 4m x 6m
- Edge conditions = s.s. slab on all the sides without any provision for torsion at corners.
- Materials : M20 grade concrete & Fe 415 HYSD bars.

Step: 1 - check for slab

$$\frac{\text{longer span}}{\text{shorter span}} = \frac{6}{4} = 1.5 < 2, \text{ hence it is two way slab.}$$

Step: 2 - Design constants

$$k = \frac{m \cdot \sigma_{cbc}}{m \cdot \sigma_{cbc} + \sigma_{st}} = 0.288$$

$$j = 1 - \frac{k}{3} = 0.904$$

$$Q = \frac{\sigma_{cbc}}{2} k \cdot j = 0.911$$

Step: 3 - Dimensions of slab

$$\frac{\text{Span}}{\text{overall depth}} = 35 \times 0.8 \text{ Fe 415}$$

IS: 456: 2000 P.No: 39

For HYSD Fe 415 / For Fe 250  
 S.S → 35 x 0.8 / S.S → 35  
 Common → 40 x 0.8 / Common → 40

Take shorter span as the main reinforcement is placed along the short span only.

$$\frac{1000}{D} = 28$$

$$D = 142.85 \text{ mm}$$

$$D = 145 \text{ mm}$$

$$d = D - \frac{\phi}{2} - \text{nominal cover}$$

↑ P.No: 47

take 10mm  $\phi$  bar

$$d = 145 - \frac{10}{2} - (20 - 5)$$

$$d = 125 \text{ mm}$$

Step: 4 - Effective span

(i) c/c of supports =  $4 + (0.15 + 0.15) = 4.30 \text{ m}$

(ii) clear span + Eff. depth =  $4 + 0.125 = 4.125 \text{ m}$  } Take lesser value

$$L_{\text{eff}} = 4.125 \text{ m.}$$

Step: 5 - Load calculation

$$L.L = 2 \times 1 = 2 \text{ kN/m}$$

$$D.L = b D \gamma = 1 \times 0.145 \times 25 = 3.625 \text{ kN/m}$$

$$F.F.L = 2.25 \times 1 = 2.25 \text{ kN/m.}$$

$$\therefore \text{Total load} = 7.875 \text{ kN/m}$$

Step: 6 - B.M

P.No: 91

$$M_x = \alpha_x \cdot W \cdot l_x^2$$

$$\alpha_x = \frac{l_y}{l_x} = \frac{6}{4} = 1.5$$

From Table: 27 ; P.No: 91

$$\alpha_x = 0.104$$

$$l_x = 4.125 \text{ m}$$

$$\therefore M_x = 13.93 \text{ kN.m}$$

$$M_y = \alpha_y \cdot W \cdot l_x^2$$

$$\alpha_y = \frac{l_y}{l_x} = 1.5$$

From IS: 456-2000 ; P.No: 91  
table: 27

$$\alpha_y = 0.046$$

$$M_y = 6.16 \text{ kN.m.}$$

Step: 7 - Check for eff. depth

$$d_{req} = \sqrt{\frac{M_x}{Qb}} = 123.65 \text{ mm}$$

$$d_{pro} = 125 \text{ mm}$$

$d_{req} < d_{pro}$ . Hence adequate.

$$Q = 0.911$$
$$b = 1000$$

$$d_{req} = \sqrt{\frac{M_y}{Q \cdot b}} = 82.23 \text{ mm}$$

$d_{req} < d_{pro}$ , Hence adequate.

Step: 8 - Reinforcement details.

(i) For shorter span

$$j = 0.904$$
$$\sigma_{st} = 230 \text{ N/mm}^2$$

$$A_{stx} = \frac{M_x}{\sigma_{st} \cdot j \cdot d_x}$$

$$= 535.97 \text{ mm}^2$$

Assume 10 mm  $\phi$  bar

$$\text{Spacing } S = 1000 \frac{a_{st}}{A_{st}} = 146.53$$

$$S \approx 145 \text{ mm}$$

Provide 10mm # @ 145mm/c.

(ii) For longer span

$$A_{sty} = \frac{M_y}{\sigma_{st} \cdot j \cdot d_y}$$

$$= 257.62 \text{ mm}^2$$

$$S = 1000 \frac{a_{st}}{A_{st}} = 300 \text{ mm}$$

Provide 10 mm # @ 300 mm/c

1. Distinguish b/w under reinforced and over reinforced sections.

Under reinforced section

Over reinforced section

\* Strain in steel at failure is above the yield point strain

\* Strain in steel at failure is below the yield point strain

\* This failure is known as tension failure

\* Compression failure.

2. How do you check the deflection for various end conditions of the beams?

IS: 456-2000; clause: 23.2.1

\* vertical deflection limits - assumed to be satisfied, provided span to depth ratio are not greater than the values of

\* Basic values of span to eff. depth ratio - for span up to 10m

cantilever beam	7
Simply supported	20
continuous	26.

\* For span above 10m

\* multiplied by  $\frac{10}{\text{span}}$  in meter except for cantilever in which case deflection calculations should be made.

3. Compute the area of reinforcement required for a balanced section of width 200mm and effective depth 425mm as per limit state design. Use M25 grade concrete and Fe415 grade steel. Use Design aids.

For balance section  $\frac{x_u}{d} = \frac{x_{u\max}}{d} = 0.48$

IS: 456-2000; clause: 38.1

$$\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} \cdot b \cdot d}$$

IS: 456-2000; clause: G-1.1

Fe415 steel grade

$$A_{st} = 1017 \text{ mm}^2$$

4. What is meant by doubly reinforced beam? (Or) Enumerate doubly reinforced section.

\* A singly reinforced section has a limiting value of moment of resistance, corresponding to limiting value of steel reinforcement.

\* If the applied moment ( $M_u$ ) is larger than  $M_u$  limit, two alternatives will be available

- \* to increase the depth of the section
- \* to provide reinforcement

\* In many cases, the max. value of depth of the



Apr/May - 2017

①

Design a rectangular RC beam in flexure and shear when it is simply supported on masonry walls 300 mm thick and 5m apart (centre to centre) to support a distributed live load of 8 kN/m and a dead load of 6 kN/m in addition to its own weight. Materials used are M20 grade of concrete and Fe415 steel bars. Adopt working stress method of design.

Data

- concrete M20 ;  $\sigma_{cbc} = 7 \text{ N/mm}^2$
- Steel Fe415 ;  $\sigma_{st} = 230 \text{ N/mm}^2$
- wall thick (b) = 300 mm
- L = 5m = 5000 mm

Load calculation:-

- L.L = 8 kN/m
- D.L = 6 kN/m

$\therefore$  Total load = 14 kN/m

B.M. calculation

$$M_R = \frac{WL^2}{8} = +3.75 \text{ k.N.m}$$

Modular ratio

$$m = \frac{280}{3 \sigma_{cbc}} = 13.33$$

Critical N.A. depth

$$x_c = \frac{m \cdot \sigma_{cbc} \cdot d}{m \cdot \sigma_{cbc} + \sigma_{st}} = 0.29 d$$

Assume Balanced section  $\therefore x_a = x_c$

To find eff. depth (d)  $\therefore M_R = M$

$$M_R = \frac{\sigma_{cbc}}{2} \cdot b \cdot x_a \left[ d - \frac{x_a}{3} \right]$$

$$d = 282 \text{ mm} \approx 300 \text{ mm.}$$

$x_a = x_c = 0.29 d = 87 \text{ mm}$

A<sub>st</sub>

$$M_R = \sigma_{st} \cdot A_{st} \left[ d - \frac{x_a}{3} \right]$$

$$A_{st} = 701.91 \text{ mm}^2$$

2

Assume 16 mm  $\phi$  rod.

$$\text{No. of bars } n = \frac{A_{st}}{A_{st} \phi} = \frac{701.91}{\pi \times 16^2 / 4} = 3.49 \approx 4 \text{ NOS.}$$

Hence provide 4 NOS of 16 mm #.

2. Design the roof slab for a Hall size 4m x 10m by working stress method using M20 concrete and Fe415 steel. The slab simply resting on 230mm thick brick walls all around. Take the live load on the slab as  $1.5 \text{ kN/m}^2$  and finish load as  $2.25 \text{ kN/m}^2$ .

Given

$$\text{concrete grade M20} \rightarrow \sigma_{cbc} = 7 \text{ N/mm}^2$$

$$\text{steel grade Fe415} \rightarrow \sigma_{st} = 230 \text{ N/mm}^2$$

$$\text{modular ratio } (m) = \frac{280}{3 \sigma_{cbc}} = 13.33$$

$$\text{Critical neutral axis depth } (x_c) = \frac{m \cdot \sigma_{cbc} \cdot d}{m \cdot \sigma_{cbc} + \sigma_{st}} = 0.29d$$

Assume balanced section,  $\therefore x_a = x_c$   
 $M_R = \text{MAX. BM.}$

B.M. calculation

$$M = \frac{W l^2}{8}$$

$$M = 8.39 \text{ kN.m.}$$

Load calculation

$$\text{Total load } (W) = \text{L.L.} + \text{F.L.}$$

$$= 1.5 + 2.25$$

$$W = 3.75 \text{ kN/m}^2$$

eff. span

$$l_{\text{eff}} = L_0 + \frac{t}{2} + \frac{t}{2} = 4.23 \text{ m}$$

To find eff. depth of slab (d)

$$M_R = \frac{\sigma_{cbc}}{2} \cdot b \cdot x_a \left[ d - \frac{x_a}{3} \right]$$

$$d = 95 \text{ mm} \approx 100 \text{ mm}$$

$$D = d + \frac{\phi}{2} + \text{clear cover}$$

$$D = 125 \text{ mm}$$

$$b = 1000 \text{ mm}$$

$$x_c = x_a = 0.29d$$

$$\text{Assume } \phi = 12 \text{ mm bar}$$

$$\text{clear cover} = 20 \text{ mm}$$

A<sub>st</sub>

$$M_R = \sigma_{st} \cdot A_{st} \left[ d - \frac{x_a}{3} \right]$$

$$A_{st} = 403.8 \text{ mm}^2$$

$$\text{No. of bars per m length} = \frac{A_{st}}{A_{\phi}} = 3.57 \approx 4 \text{ NOS.}$$

$$x_a = 0.29d = 29 \text{ mm}$$

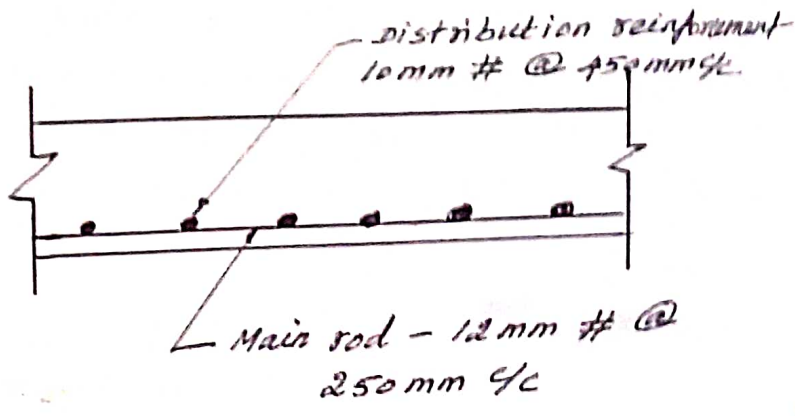
spacing =  $\frac{1000}{4} = 250 \text{ mm}$

(vi)  $S = \frac{A_{\phi} \times 1000}{A_{st_{act}}}$

Check for spacing

(i)  $\frac{1000}{4} = 250 \text{ mm}$

(ii)  $\neq 3d = 3 \times 100 = 300 \text{ mm}$



Distribution Reinforcement

Assume 10mm #

spacing max.  $5d = 5 \times 100 = 500 \text{ mm}$

$\therefore S = 450 \text{ mm}$

NOV/DEC - 2016

1 Explain the codal recommendations for limit state design? state their significance.

Refer. IS: 456 - 2000 - P. NO: 67 to 76

\* Safety and serviceability requirements

- \* Limit state of collapse
- \* Limit state of serviceability
  - (a) Deflection
  - (b) Cracking

\* Characteristic and design values and partial safety factors

- \* Characteristic strength of materials
- \* Characteristic Loads
- \* Design Values
  - Materials
  - Loads
  - consequences of Attaining LS
- \* Partial Safety factors
  - Load
  - Material

\* Limit state of collapse

- \* Flexure
- \* Compression
- \* Shear
- \* Torsion

\* Limit state of serviceability: Deflection — Flexural member

\* Limit state of serviceability: Cracking — Flexural members, Compression Members

4

2) Design a rectangular section for a simply supported reinforced concrete beam of effective span of 4m carrying a concentrated load of 35 kN at its midspan. The concrete to be used of grade M20 and the reinforcement consists of Fe415 steel bars.

(i) self weight of beam is ignored

(ii) self weight of beam is considered.

Use working stress method.

Ans

(i)

$$\sigma_{cbc} = 7 \text{ N/mm}^2$$

$$\sigma_{st} = 230 \text{ N/mm}^2$$

$$m = \frac{280}{3\sigma_{cbc}} = 13.33$$

Assume  $b = \frac{d}{2}$

$$x_c = \frac{m \cdot \sigma_{cbc} \cdot d}{m \cdot \sigma_{cbc} + \sigma_{st}} = 0.29d$$

Assume balanced section  $x_a = x_c$

(i) self wt of beam is ignored

(ii) self wt of beam is considered

Load calculation

$$W = 35/4 = 8.75 \text{ kN/m}$$

$$B.M = M = \frac{Wl^2}{8} = \frac{8.75 \times 4^2}{8} = 17.5 \text{ kN.m}$$

To find eff. depth (d)

$$M_R = \frac{\sigma_{cbc} \cdot b \cdot x_a}{2} \left[ d - \frac{x_a}{3} \right]$$

$$x_a = 0.29d ; b = \frac{d}{2}$$

$$d = 336.6 \text{ mm} \approx 350 \text{ mm}$$

$$\therefore b = \frac{d}{2} = 175 \text{ mm} \approx 230 \text{ mm}$$

A<sub>st</sub>

$$M_R = \sigma_{st} \cdot A_{st} \cdot \left[ d - \frac{x_a}{3} \right]$$

$$x_a = 0.29d = 101.5 \text{ mm}$$

$$A_{st} = 240.07 \text{ mm}^2$$

No. of bars

$$n = \frac{A_{st}}{A_\phi}$$

$\therefore$  Adopt beam size 230 mm x 350 mm

Provide 3 - 12 mm #

Load calculation

$$W = 8.75 \text{ kN/m}$$

$$\text{Self wt} = 1 \times 0.35 \times 25 = 8.75 \text{ kN/m}$$

$$\therefore \text{Total load} = 17.5 \text{ kN/m}$$

B.M

$$M = \frac{Wl^2}{8} = 35 \text{ kN.m}$$

A<sub>st</sub>

$$M_R = \sigma_{st} \cdot A_{st} \cdot \left[ d - \frac{x_a}{3} \right]$$

$$A_{st} = 481.3 \text{ mm}^2$$

No. of bars

$$n = \frac{A_{st}}{A_\phi}$$

Provide 3 nos - 16 mm #

A beam simply supported over an effective span of 8m carries a live load of 15kN/m. Design the beam, using M20 concrete and Fe415 grade steel. Keep the width equal to half the effective depth. Use working stress method of design.

Given data

$L = 8m$   
 Live load = 15kN/m  
 $f_y = 415 \text{ N/mm}^2$ ;  $\sigma_{st} = 230 \text{ N/mm}^2$   
 $f_{ck} = 20 \text{ N/mm}^2$ ;  $\sigma_{cbc} = 7 \text{ N/mm}^2$   
 $b = \frac{d}{2}$

Design parameters

$m = \frac{280}{3\sigma_{cbc}} = 13.33$   
 $k = \frac{m \cdot \sigma_{cbc}}{m \cdot \sigma_{cbc} + \sigma_{st}} = 0.289$   
 $j = 1 - \frac{k}{3} = 0.904$   
 $Q_r = \frac{\sigma_{cbc}}{2} k \cdot j = 0.914$

Step: 1 - Dimensions

Eff. depth ( $d$ ) =  $\frac{\text{span}}{10} = 800 \text{ mm}$   
 overall depth ( $D$ ) =  $d + d'$   
 $= 850 \text{ mm}$   
 Eff. span = 8m = 8000 mm  
 width of beam ( $b$ ) =  $\frac{d}{2} = 400 \text{ mm}$

Step: 2 - Load calculation

self wt of beam =  $b \cdot D \cdot \rho$   
 $= 0.4 \times 0.85 \times 25$   
 $= 8.5 \text{ kN/m}$   
 Live load = 15kN/m  
 $\therefore$  Total load ( $W$ ) = 23.5kN/m

Step: 3 - Moment & SF

$M = \frac{Wl^2}{8} = \frac{23.5 \times 8^2}{8}$   
 $M = 188 \text{ kN.m}$   
 $V = \frac{Wl}{2} = \frac{23.5 \times 8}{2}$   
 $V = 94 \text{ kN}$

Step: 4 - Reinforcement

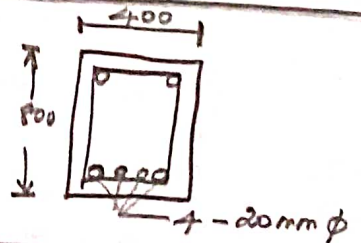
$M = \sigma_{st} \cdot j \cdot d \cdot A_{st}$   
 $A_{st} = \frac{M}{\sigma_{st} \cdot j \cdot d} = \frac{188 \times 10^6}{230 \times 0.904 \times 800}$   
 $A_{st} = 1135.27 \text{ mm}^2$

No. of bars, Assume 20mm  $\phi$

$n = \frac{A_{st}}{A_{\phi}} = \frac{1135.27}{\pi \times 20^2 / 4} = 8.6$   
 say 4 nos.

Spacing

$S = \frac{1000 A_{\phi}}{A_{st}} = \frac{1000 \times \pi \times 20^2 / 4}{4 \times \pi \times 20^2 / 4}$   
 $= 250 \text{ mm}$   
 Provide 4 - 20mm  $\phi$  @ 250mm c/c



Nov/Dec - 2017

1. Design a simply supported reinforced concrete beam to carry a B.M of 50kN.m as doubly reinforced section by working stress design. Keep the width is equal to half the effective depth. M25 & Fe415 grade of concrete & steel

Given data

$M = 50 \text{ kN.m}$   
 $b = \frac{d}{2}$   
 $\sigma_{cbc} = 8.5 \text{ N/mm}^2$   
 $\sigma_{st} = 230 \text{ N/mm}^2$

Design parameters

$m = \frac{280}{3\sigma_{cbc}} = 10.98$   
 $k = \frac{m \cdot \sigma_{cbc}}{m \cdot \sigma_{cbc} + \sigma_{st}} = 0.289$   
 $j = 1 - \frac{k}{3} = 0.904$   
 $Q_r = \frac{\sigma_{cbc}}{2} j \cdot k = 1.11$

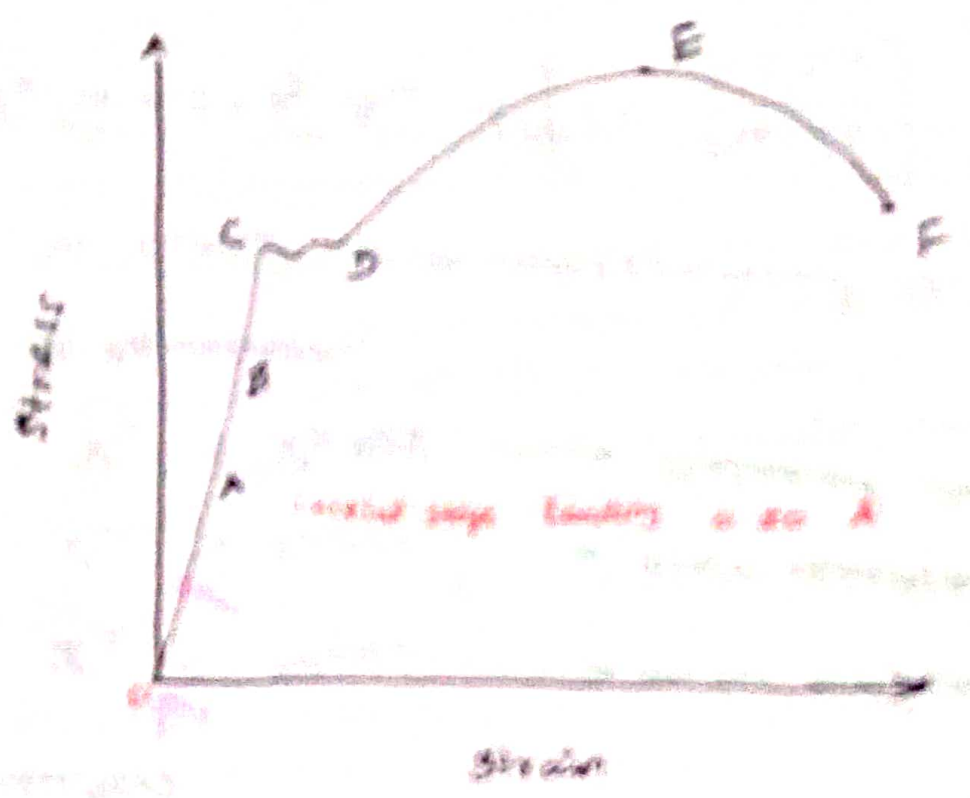
Depth & breadth

Assume balanced section  
 $M = Q_r b d^2$   
 $50 \times 10^6 = 1.11 \cdot \frac{d}{2} \cdot d^2$   
 $\frac{50 \times 10^6 \times 2}{1.11} = d^3$   
 $\therefore d = 448 \text{ mm}$

for doubly reinforced section

$\therefore d = 400 \text{ mm}$   
(400mm Assume)  
 $D = d + d' = 450 \text{ mm}$   
 $b = \frac{d}{2} = 200 \text{ mm}$

# Stress-strain behaviour



- A — limit of proportionality
- B — elastic limit
- OAB — obey Hooke's law
- elastic stage
- C → yield stress
- CD — plastic yielding (an increase in stress)
- D — causes large strain
- E — max. ultimate stress of ultimate load
- F — breaking stress

-II - Limit state Method

Beam - singly Reinforced - TYPE - I - To find M.R

1)  $A_{st} = \text{No. of bar} \times \frac{\pi d^2}{4}$  dia of bar

2) Depth of Neutral axis ( $x_u$ )

$$\frac{x_u}{d} = \frac{0.87 f_y \cdot A_{st}}{0.36 f_{ck} \cdot b d}$$

IS: 456: 2000 - P. NO: 96  
G 1.1.1. (a)

3) Limiting value of neutral axis ( $x_{u \max}$ )

$M_u \text{ limit} = 0.148 f_{ck} b d^2$   $\frac{x_{u \max}}{d} = 0.53$

Fe 250

$M_u \text{ limit} = 0.138 f_{ck} b d^2$   $\frac{x_{u \max}}{d} = 0.48$

Fe 415

$M_u \text{ limit} = 0.133 f_{ck} b d^2$   $\frac{x_{u \max}}{d} = 0.46$

Fe 500

IS: 456: 2000  
P. NO: 70

$f_y \frac{x_{u \max}}{d}$

4) Comparison

$x_u < x_{u \max}$   $\rightarrow$  under reinforced section

$x_u > x_{u \max}$   $\rightarrow$  over reinforced section

$x_u = x_{u \max}$   $\rightarrow$  Balanced section

5) Moment of Resistance

(a)  $x_u < x_{u \max}$   $\rightarrow$  under reinforced section

$$M_u = 0.87 f_y \cdot A_{st} \cdot d \left[ 1 - \frac{A_{st} \cdot f_y}{b d f_{ck}} \right]$$

IS: 456: 2000  
P. NO: 96  
G. 1.1.1. (b)

(b)  $x_u = x_{u \max}$   $\rightarrow$  Balanced section

$$M_{u \text{ limit}} = 0.36 \frac{x_{u \max}}{d} \left[ 1 - 0.42 \frac{x_{u \max}}{d} \right] b d^2 f_{ck}$$

P. NO: 96  
G. 1.1.1. (c)

(c)  $x_u > x_{u \max}$   $\rightarrow$  over reinforced section  $\rightarrow$  Redesigned

$$M_{u \text{ limit}} = 0.36 \frac{x_{u \text{ max}}}{d} \left[ 1 - 0.42 \frac{x_{u \text{ max}}}{d} \right] b d^2 f_{ck}$$

P.NO: 96  
G.1.1(C)

(c)  $x_u > x_{u \text{ max}} \rightarrow$  over reinforced section  $\rightarrow$  Redesigned

Problem: A rectangular reinforced concrete beam has a width of 200 mm & is reinforced with 2 bars of 20 mm diameter at an effective depth of 400 mm. If M20 grade concrete and Fe 415 HYSD bars are used. Estimate the ultimate moment of resistance of the section.

Ans:  $\frac{x_u}{d} = 0.39$  ;  $A_{st} = 628 \text{ mm}^2$  ;  $\frac{x_{u \text{ max}}}{d} = 0.48$   
 $M_u = 76 \text{ KN}\cdot\text{m}$

$x_u < x_{u \text{ max}}$  (under reinforced)

Type - II - To find  $A_{st}$  & No. of bars.

Given:  $b, d, f_{ck}, f_y$

(1) Assume Balanced section  $\therefore x_u = x_{u \text{ max}}$

$M_{u \text{ limit}}$	$f_y$	$x_{u \text{ max}}/d$	} $\rightarrow$ IS : 456 : 2000 - P.NO: 70
$0.148 f_{ck} b d^2$	250	0.53	
$0.138 f_{ck} b d^2$	415	0.48	
$0.133 f_{ck} b d^2$	500	0.46	

(2) Limiting value of Moment of resistance

$$M_{u \text{ limit}} = 0.36 \frac{x_{u \text{ max}}}{d} \left[ 1 - 0.42 \frac{x_{u \text{ max}}}{d} \right] b d^2 f_{ck}$$

P.NO: 96  
G.1.1(C)

(3)  $A_{st}$

$$\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b d}$$

IS : 456 : 2000, P.NO: 96  
G.1.1(a)

$A_{st} = ?$

(4) No. of rods

$$n = \frac{A_{st}}{\pi d^2 / 4}$$

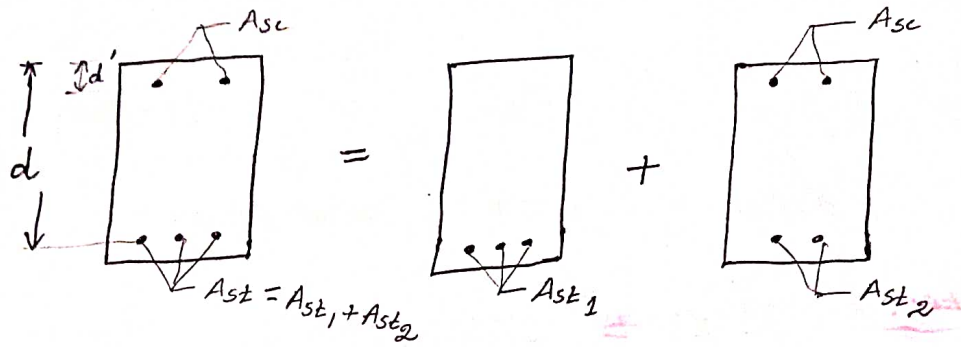
dia of bar

5) Check  $\frac{A_{st \text{ min}}}{A_{st}} = \frac{0.85}{f_y}$   $\angle A_{st} < A_{st \text{ max}}$   
P.NO: 46447

(6)  $A_{st \text{ max}} = 0.04 b D$   
 $D = d + d'$



## Doubly Reinforced Beam.



$$A_{st2} = A_{sc}$$

$$M_u = M_{u1} + M_{u2}$$

$$M_{u2} = f_{sc} \cdot A_{sc} \cdot (d - d')$$

- i) A doubly reinforced section of size  $300\text{ mm} \times 600\text{ mm}$ . The beam is reinforced with 2 bars of  $16\text{ mm}$  diameter in the compression side and 4 bars of  $20\text{ mm}$  dia in the tension side. The concrete is of M20 grade and Fe415 HYSD bars are used. An effective cover of  $50\text{ mm}$  is provided on both top and bottom. Find the moment of resistance.

Step: 1 - Given data:

$$b = 300\text{ mm}$$

$$D = 600\text{ mm}$$

$$d_{\text{eff}} = 50\text{ mm}$$

$$\therefore d = 600 - 50 = 550\text{ mm}$$

$$f_{ck} = 20\text{ N/mm}^2$$

$$f_y = 415\text{ N/mm}^2$$

$$A_{st} = 4 - 20\text{ mm} \# = 4 \times \pi \times 20^2 / 4$$

$$A_{sc} = 2 - 16\text{ mm} \# = 2 \times \pi \times 16^2 / 4 = A_{st2}$$

Step: 2 - Limiting Neutral axis depth ( $x_{u,lim}$ )

FOR Fe415 grade steel,

$$\frac{x_{u,max}}{d} = 0.48 \quad ; \quad x_{u,max} = 0.48d$$

$$x_{u,max} = 0.48 \times 550 = 264 \text{ mm.}$$

Step: 3 - stresses in steel

(i) Stress in compression steel,

$$f_{sc} = E_{sc} \cdot \epsilon_s = \left[ \frac{0.0035 (x_{u,max} - d')}{x_{u,max}} \right] E_s$$

$$= \frac{0.0035 \times (264 - 50)}{264} \times 2 \times 10^5$$

$$f_{sc} = 567.4 \text{ N/mm}^2$$

(ii)  $0.87 f_y = f_{sc}$

$$f_{sc} = 0.87 \times 415 = 339.06 \text{ N/mm}^2$$

check

$$f_{sc} > 0.87 f_y$$
$$567.4 > 339.06 \text{ N/mm}^2$$

∴ Take Least value

$$\therefore f_{sc} = 0.87 f_y = 339.06 \text{ N/mm}^2$$

Step: 4 - Area of reinforcement ( $A_{st}$ )

$$A_{st} = A_{st1} + A_{st2}$$

$$(4 \times \pi \times 20^2 / 4) = A_{st1} + (2 \times \pi \times 16^2 / 4)$$

$$\therefore A_{st1} = (4 \times \pi \times 20^2 / 4) - (2 \times \pi \times 16^2 / 4)$$

$$A_{st2} = A_{sc}$$

$$A_{st1} = 854.08 \text{ mm}^2$$

Step: 5 - actual neutral axis depth ( $x_u$ )

$$x_u = \frac{0.87 f_y A_{st1}}{0.36 f_{ck} b} = \frac{0.87 \times 415 \times 854.08}{0.36 \times 20 \times 300}$$

$$x_u = 142.76 \text{ mm}$$

Step: 6 - check for under (or) over reinforced section

$$x_u = 142.76 \text{ mm} < x_{u\max} = 264 \text{ mm}$$

Hence, under reinforced section.

Step: 7 - Moment of Resistance ( $M_r$ )

$$M_u - M_{u\lim} = f_{sc} A_{sc} (d - d')$$

$$M_{u\lim} = 0.36 \frac{x_{u\max}}{d} \left(1 - 0.42 \frac{x_{u\max}}{d}\right) f_{ck} b d^2$$

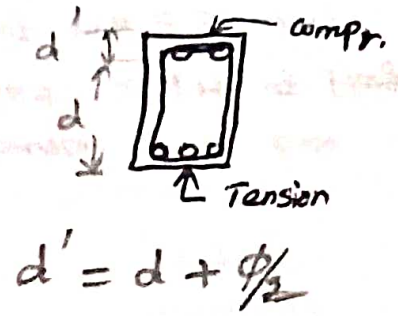
$$= 0.36 \times 0.48 \left(1 - 0.42 \times 0.48\right) 20 \times 300 \times (500)^2$$

$$= 1733 \text{ kNm}$$

IS: 456-2000  
P. NO: 96  
A-1-1(a)

# Doubly Reinforced Beam

\* The beams having steel reinforcement both on tensile and compression zones (forces) are known as doubly reinforced beam.



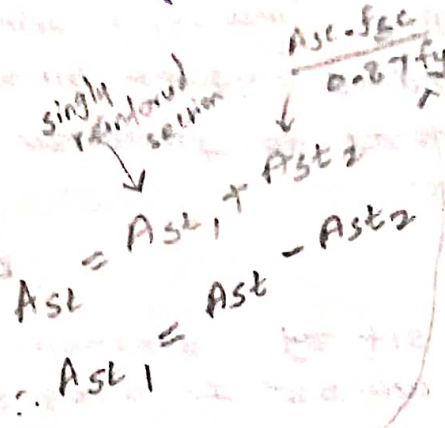
Procedure:

1)  $A_{st}, A_{sc}$

2)  $\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b d}$

3)

$\frac{x_u}{d}$	$f_y$	$x_{u, max}/d$
	250	0.53
	415	0.48
	500	0.46



IS: 456: 2000 - P NO: 96

P. NO: 70

4) To find design stress  $f_{sc}$  (or)  $\sigma_{sc}$

$2 \times 10^5 \text{ N/mm}^2 \rightarrow E_{sc} = \frac{\sigma_{sc}}{\epsilon_{sc}}$

Stress in steel in compr. zone  
Strain in steel comp. zone

$\epsilon_{sc} = 0.0035 \left( \frac{x_{u, max} - d'}{x_{u, max}} \right)$

P. NO: 96, Gr. 1.2

$\sigma_{sc} = E_{sc} \cdot \epsilon_{sc}$

Take least Value

$\sigma_{sc} = \frac{f_y}{1.15}$  (or)  $\sigma_{sc} = 0.87 f_y$

check

$x_u < x_{u, max}$  (under reinforced section)

$x_u > x_{u, max}$  (over reinforced section) Redesign

$x_u = x_{u, max}$  (Balanced section)

(6) Mu (or) M.R

$x_u < x_{u \max}$  (under)

$$M_u = 0.87 f_y A_{st} d \left[ 1 - \frac{A_{st} f_y}{f_{ck} b d} \right]$$

P.No: 96 G.I.1.(b)

$x_u = x_{u \max}$  (Balanced section)

$$M_{u \lim} = 0.36 \frac{x_{u \max}}{d} \left[ 1 - 0.42 \frac{x_{u \max}}{d} \right] b d^2 f_{ck}$$

P.No: 96; G.I.1.(c)

(7) Moment taken by compression zone

$$M_u - M_{u \lim} = M_{u_2}$$

P.No: 96, G.I.2

$$M_{u_2} = f_{sc} A_{sc} (d - d')$$

$$M_u = M_{u_2} + M_{u \lim}$$

Problem: A doubly reinforced concrete beam having a rectangular section 250mm wide & 540mm overall depth is reinforced with 2 bars of 12mm diameter in the compression side & 4 bars of 20mm diameter in the tension side. The eff. cover to bars is 40mm, using M20 grade concrete & Fe415 HYSD bars, Estimate the flexural strength of the section using IS:456:2000 - code recommendations.

$D = 540$ ;  $d' = 40$ ;  $d = 500$

Ans:  $A_{st} = 1256 \text{ mm}^2$ ;  $A_{sc} = 226 \text{ mm}^2$ ;  $A_{st} = 1030 \text{ mm}^2$

\*  $x_u = 206.6 \text{ mm}$ ; \*  $x_{u \max} = 240 \text{ mm}$ ; under reinforced,

$M_u = 191 \text{ kN.m}$

$b = 250 \text{ mm}$ ;  $d = 500 \text{ mm}$ ;  $D = 540 \text{ mm}$ ;  $d' = 40 \text{ mm}$

$$A_{st} = 4 \times \pi \times 20^2 / 4 = 1256 \text{ mm}^2$$
;  $A_{sc} = 2 \times \pi \times 12^2 / 4 = 226 \text{ mm}^2$

$$x_{u \lim} = 0.48 d = 0.48 \times 500 = 240 \text{ mm}$$

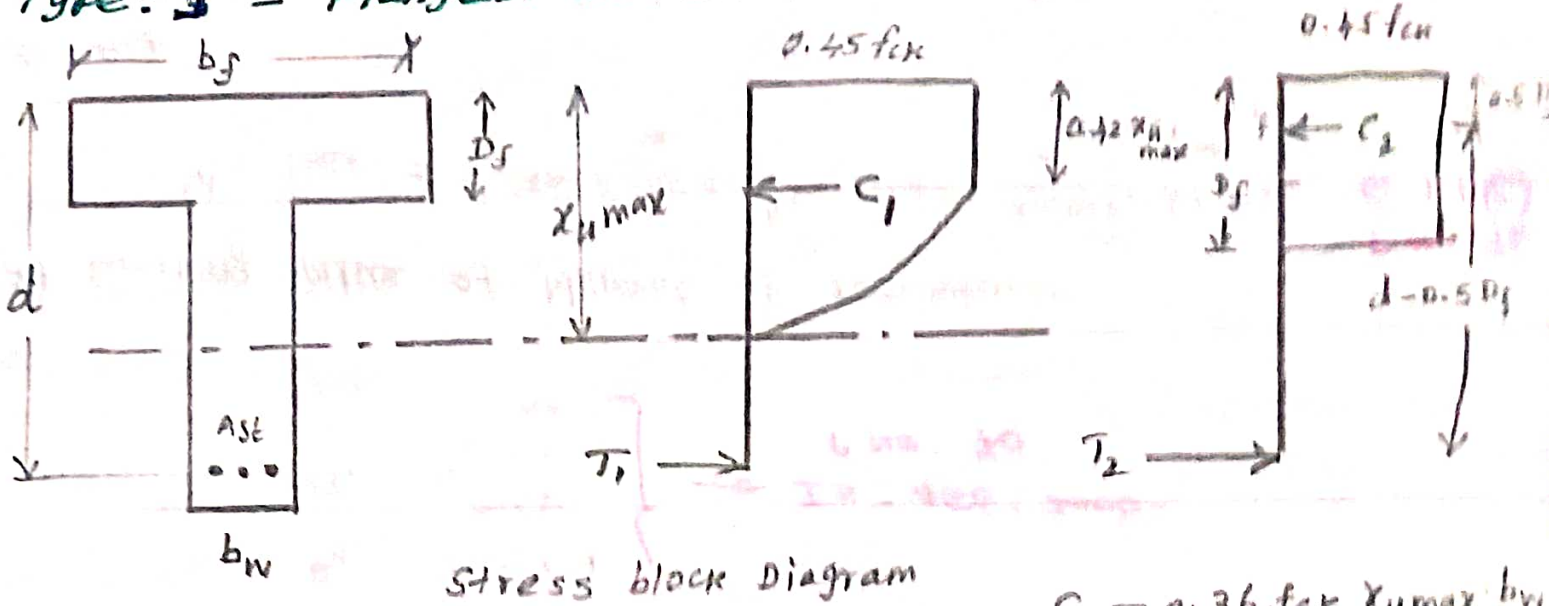
$$f_{sc} = \epsilon_s \cdot E_s = 0.0035 \left[ \frac{240 - 40}{240} \right] \times 2 \times 10^5 = 583 \text{ N/mm}^2 \neq 0.87 f_y = 360$$

$A_{st_2} = \frac{f_{sc} A_{sc}}{0.87 f_y} = 226 \text{ mm}^2$

$A_{st_1} = A_{st} - A_{st_2} = 1256 - 226 = 1030 \text{ mm}^2$

## Unit - II - Flanged Beam

Type: 3 - Flanged section - T-Beam.



$$C_1 = 0.36 f_{ck} x_{u,max} b_w$$

$$C_2 = 0.45 f_{ck} D_f (b_f - b_w)$$

(1)  $A_{st}$

(2) Depth of neutral axis ( $x_u$ )

$$b = b_f$$

$$\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} \cdot b d}$$

IS: 456: 2000; P.No: 96  
Gr. 1.1. (a)

(3)  $\frac{x_{u,max}}{d}$

$f_y$	$x_{u,max}/d$
Fe 250	0.53
415	0.48
500	0.46

P.No: 70

(4) check

case: I - N.A lies within the flange portion

P.No: 96 Gr. 2.1

$x_u < D_f$  if it is ok, then

M.R

Gr. 1.1 (b)

Gr. 1.1 (c)

$x_u < x_{u,max}$  Under reinforced  
 $x_u = x_{u,max}$  Balanced  
 $x_u > x_{u,max}$  Over

$b = b_w$

not satisfy case I, then case: II

Case: II

N.A lies outside the flange portion  
 $\frac{D_f}{d} \geq 0.2$

P.NO: 96, G.2.2

$$M_u = 0.36 \frac{x_{u \max}}{d} \left[ 1 - 0.42 \frac{x_{u \max}}{d} \right] f_{ck} b_w d^2 + 0.45 f_{ck} (b_f - b_w) D_f \left( d - \frac{D_f}{2} \right)$$

(P.NO: 96 G.2.2)

Case: III)  $I_f$   $x_u > D_f$  ;  $D_f/d > 0.2$

P.NO: 97, G.2.3.1

$$M_u = 0.36 \frac{x_{u \max}}{d} \left[ 1 - 0.42 \frac{x_{u \max}}{d} \right] f_{ck} b_w d^2 + 0.45 f_{ck} (b_f - b_w) y_f \left( d - \frac{y_f}{2} \right)$$

$$y_f = (0.15 x_u + 0.65 D_f) \neq D_f$$

Case II (c)

$$x_{u \max} > x_u > D_f \text{ \& } \frac{D_f}{x_u} \neq 0.43$$

P.NO: 97  
G.2.3

$$M_u = 0.36 \frac{x_{u \max}}{d} \left[ 1 - 0.42 \frac{x_{u \max}}{d} \right] f_{ck} b_w d^2 + 0.45 f_{ck} (b_f - b_w) y_f \left( d - \frac{y_f}{2} \right)$$

$$y_f = 0.15 x_u + 0.65 D_f$$

Problem: 2 Calculate the ultimate flexural strength of a T-beam section having the following sectional properties.

width of flange = 1200 mm ; Depth of flange = 120 mm ;

width of rib = 300 mm ; Effective depth = 600 mm ;

Area of tensile steel = 8 bars of 25 mm diameter.

Materials: M20 grade concrete. Fe 415 HYSD bars.

Problem 1 → Determine the moment of resistance of a T-section having the following section properties: width of flange = 2500 mm ; Depth of flange = 150 mm ; width of rib = 300 mm ; Effective depth 800 mm ;  $A_{st} = 8$  bars of 25 mm # ;

Material: Fe 415, M20

Ans  $A_{st} = 3926$  ;  $x_u = 78.4$

$x_u < D_f$  ;  $M_u = 1089 \text{ kNm}$

## Development Length ( $L_d$ )

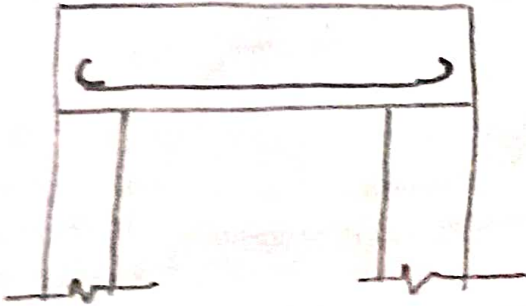
IS: 456: 2000 ; P. NO: 42

CI: 26.2.1

Tension

$$L_d = \frac{\phi \sigma_s}{4 \tau_{bd}}$$

$L_d$



$\phi \rightarrow$  dia of bar

$\sigma_s = 0.87 f_y \rightarrow$  stress in bar

$\tau_{bd} \rightarrow$  design bond stress

$\leftarrow$  26.2.1.1 ; P. NO: 43

based on grade of concrete.

$$L_d \text{ in comp} = \frac{\phi \sigma_s}{4 \tau_{bd} \cdot (1.25)}$$

- 1) A cantilever beam carrying a udl has a breadth of 150 mm & effective depth of 260 mm, the reinforcement consist of 4 bars of 16 mm dia. The factored total load is 75 kN. Calculate the max. bond stress & anchorage length required. Assume,  $f_{ck} = 30 \text{ N/mm}^2$ ;  $F_y = 415 \text{ N/mm}^2$ .

Given	$A_{st}$	Bond stress	Development length
$f_{ck}$	$d = 16$	$U = \frac{V}{\sum_0 \cdot d}$ eff. depth	$L_d = \frac{\sigma_s \phi}{4 \tau_{bd}}$
$f_y$	$\frac{4 \times \pi d^2}{4}$	Perimeter $\sum_0$	P. NO: 42
$b = 150$	$A_{st} = 804 \text{ mm}^2$	$\sum_0 = \pi \cdot 16 \cdot 4$	$\tau_{bd} = 1.5 + 60\%$ P. NO: 43
$d = 260 \text{ mm}$		$= \pi \times 16 \times 4$	$= 1.5 + \frac{60}{100} \times 1.5$
$V = 75 \text{ kN}$		$= 201.6 \text{ mm}$	$= 2.4$
		$U = 1.43 \text{ N/mm}^2$	$L_d = 602 \text{ mm}$

Anchorage length for 16 mm  $\phi$  bar = 602 mm

- 2) A S.S beam 6m its span it carries a characteristic load of 60 kN/m. If ~~the~~ 6 nos of 20 mm  $\phi$  bars are provided at the centre of span & 4 nos of these bars are continued in to the support. Check the development length at support. Assume M15 & Fe 415 grade steel.



Given  
 $f_{ck}$   
 $f_y$   
 $L = 6m$   
 Char. load =  $60 kN/m$   
 Factored load =  $1.5 \times 60 = 90 kN/m$

Max SF & BM  
 $S.F = \frac{wl}{2} = \frac{90 \times 6}{2} = 270 kN$   
 $BM = \frac{wl^2}{8} = \frac{90 \times 6^2}{8} = 405 kN \cdot m$

Moment taken by reinforcement at support  
 BM at 6 nos. of bars =  $405 kN \cdot m$   
 B.M at 4 nos. of bar =  $\frac{405}{6} \times 4 = 270 kN \cdot m$

Development length  
 $L_d = \frac{0.87 f_y \phi}{4 \tau_{bd}}$   
 P. NO: 43  
 $\tau_{bd}$  to increase 60%  
 $M_{15} = 1$   
 $\tau_{bd} = 1 + \frac{60}{100} \times 1 = 1.6 N/mm^2$   
 $L_d = 1130 mm$

Check for  $L_d$

P. NO: 44,

$$1.3 \frac{M_1}{V} + L_0$$

$$M_1 = 270 kN \cdot m$$

$$V = 270 kN$$

$$L_0 = d \text{ (or) } 12\phi$$

$d$  is not given

$$L_0 = 12\phi = 12 \times 20 = 240 mm$$

$$L_d < 1.3 \frac{M_1}{V} + L_0$$

$$1130 < 1540$$

Hence safe.

Check for the development length at support of a doubly reinforced beam  $400 \times 750 mm$  (effective), the clear span of the beam is  $5.25 m$ . The beam carries a UDL of  $46 kN/m$  (including self wt). The beam is reinforced with 8 bars of  $20 mm$  diameter (4 are bent up near support) on tension side and 4 bars of  $16 mm$  diameter on compression side. Adopt M20 & Fe415

Given  
 size of beam  $b \times d = 400 \times 750 mm$   
 clear span  $(l) = 5.25 m$   
 UDL =  $46 kN/m$   
 Bars  $\rightarrow$  8 nos -  $20 mm \phi$   
 4 nos -  $16 mm \phi$

Design Load  
 $W_u = 1.5 \times 46 = 69 kN/m$   
 Shear force  
 $V_u = \frac{W_u \cdot l}{2} = 34.13 kN$

shear

① A singly reinforced beam having a eff. depth of 600 mm & the breadth of 300 mm. It is reinforced with 3 bars of 16 mm # in the tension zone & 2 bars of 16 mm # in the compression zone. Factor shear force develop at the support is 180 kN. Use M20 & Fe 415. Design shear reinforcement.

Given data  
 $b = 300 \text{ mm}$   
 $d = 600 \text{ mm}$   
 $A_{st} = 3 \times \pi \times \frac{16^2}{4} = 603.2 \text{ mm}^2$   
 $A_{sc} = 2 \times \pi \times \frac{16^2}{4} = 402.1 \text{ mm}^2$   
 $V_u = 180 \text{ kN}$   
 $f_{ck} = 20 \text{ N/mm}^2$   
 $f_y = 415 \text{ N/mm}^2$

Nominal shear stress ( $\tau_v$ )  
 IS: 456-2000-P.NO: 72  
 $\tau_v = \frac{V_u}{bd} = 1 \text{ N/mm}^2$

Permissible shear stress ( $\tau_c$ )  
 IS: 456-2000-P.NO: 73  
 $\frac{100 A_{st}}{bd} = \frac{100 \times 603.2}{300 \times 600} = 0.34 \text{ N/mm}^2$

From Table: 20  
 $0.25 \rightarrow$   
 $0.50 \rightarrow$   
 $\tau_c = 0.4 \text{ N/mm}^2$

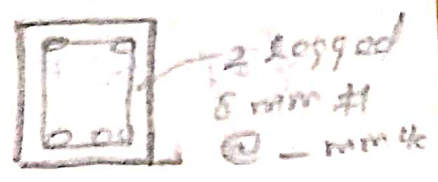
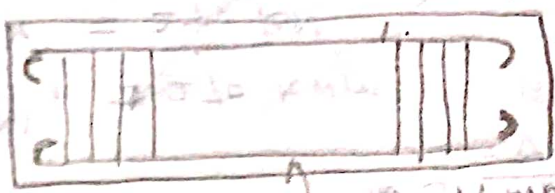
check  
 $\tau_v > \tau_c$   
 $1 > 0.4$   
 $\therefore$  shear reinforcement necessary

Design of shear reinforcement

Assume 2 legged 8 mm # vertical stirrups  
 IS: 456-2000; P.NO: 73  
 $V_{us} = V_u - \tau_c b d = 108 \text{ kN}$

Spacing  
 IS: 456-2000-P.NO: 73  
 $V_{us} = \frac{0.87 f_y A_{sv} \cdot d}{S_v}$   
 $A_{sv} = 2 \times \pi \times \frac{8^2}{4} = 56.55 \text{ mm}^2$   
 $S_v = ?$   
 2 legged 8 mm #  
 2 + 16 mm #

(ii) IS: 456-2000; P.NO: 48  
 Cl: 26.5.1.6  
 $\frac{A_{sv}}{b \cdot S_v} = \frac{0.4}{0.87 f_y}$   
 $S_v = ?$   
 Take least value.



② A RC beam having a eff. size of 350 x 550 mm is reinforced with 4 nos of 20 mm dia, use Fe 415 out of which 2 bars are crank with support. If the beam is subjected to an ultimate shear force of 400 kN. Design the shear reinforcement if required considering M20 & Fe 415.

Given data  
 $b = 350 \text{ mm}$   
 $d = 550 \text{ mm}$   
 $A_{st} = 2 \times \pi \times \frac{20^2}{4} = 628.32 \text{ mm}^2$

$V_u = 400 \text{ kN}$   
 $f_y = 415 \text{ N/mm}^2$   
 $f_{ck} = 20 \text{ N/mm}^2$

Nominal shear stress  
 P.NO: 73  
 $\tau_v = \frac{V_u}{bd} = 2.08 \text{ N/mm}^2$

Permissible shear stress ( $\tau_c$ )  
 $\frac{100 A_{st}}{bd} = 0.33$   
 $0.25 \rightarrow 0.36$   
 $0.5 \rightarrow 0.48$   
 $0.33 \rightarrow 0.4 \text{ N/mm}^2$

check

$T_v > T_c$   
Shear reinforcement necessary

Design of shear reinforcement

Assume, 2 legged 10mm vertical stirrups.

IS: 456-2000 - P No: 73

$$V_{us} = V_u - (T_c \cdot bd + 0.87 f_y A_{sv} \sin \alpha)$$

Shear resisted by concrete  $\alpha = 45^\circ$  bent up bars.  
Shear resisted by

$$V_{us} = 162.59 \text{ kN}$$

Spacing

IS: 456 - P-NO: 73

$$V_{us} = \frac{0.87 f_y A_{sv} \cdot d}{S_v}$$

$$S_v = 191.85 \text{ mm.}$$

Spacing

IS: 456-2000 P.No: 48

$$(ii) \frac{A_{sv}}{b \cdot S_v} = \frac{0.4}{0.87 f_y}$$

$$S_v = ?$$

(iii) IS: 456: 2000 - PNO: 47

$$(ii) 0.75 d$$

$$(iv) 300 \text{ mm}$$

Take least one.

Hence provide 2 legged 10mm # vertical stirrups @ 190mm

3) A RC beam has a support section with a width of 250mm and eff. depth of 500mm. The support section is reinforced with 3 bars of 20mm  $\phi$  on the tension side. 8mm dia 2 legged stirrups are provided at a spacing of 200mm centres. Using M20 + Fe415 HYSD. bars. calculate shear strength of the support section.

P.No: 73  $V_{us} = V_u - T_c \cdot bd$

Given data

$b = 250 \text{ mm}$   
 $d = 500 \text{ mm}$

$$A_{st} = 3 \times \pi \times \frac{20^2}{4} = 942 \text{ mm}^2$$

$$S_v = 200 \text{ mm}$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

$$A_{sv} = 2 \times \pi \times \frac{8^2}{4} = 100 \text{ mm}^2$$

$T_c$

IS: 456-2000, P.No: 72 Table 20

$$\frac{100 A_{st}}{bd}$$

$$T_c = 0.56 \text{ N/mm}^2$$

Shear resisted by concrete

P.No: 73  
 $V_{uc} = T_c \cdot bd = 70 \text{ kN.}$

Shear resisted by vertical links

P.No: 73  
 $V_{us} = \frac{0.87 f_y \cdot A_{sv} \cdot d}{S_v} = 90.26 \text{ kN.}$

Total Shear resistance of support section

P.No: 73  
 $V_{us} = V_u - T_c \cdot bd$   
 $\therefore V_u = V_{us} + T_c \cdot bd = 90.26 + 70 = 160.26 \text{ kN}$

Shear failure in RC members

1. Shear tension or diagonal tension failure
2. Flexure shear failure
3. Shear compression failure
4. Shear bond failure

Torsion

① A rectangular beam in a multistoried building is 250 mm wide & 500 mm deep. The section is subjected to a BM of 55 kN.m. Ultimate torsional moment 30 kN.m, Ultimate shear force 40 kN. Using M20 + Fe 415 grade. Take eff. cover is 50 mm

Given data

- $b = 250 \text{ mm}$
- $D = 500 \text{ mm}$  &  $d' = 50 \text{ mm}$
- $f_{ck} = 20 \text{ N/mm}^2$        $d = 500 - 50 = 450 \text{ mm}$
- $f_y = 415 \text{ N/mm}^2$
- $M_u = 55 \text{ kN.m}$
- $T_u = 30 \text{ kN.m}$
- $V_u = 40 \text{ kN}$

Step: 1 - Longitudinal reinforcement

IS: 456-2000; P.NO: 75; Cl: 41.4.2

$$M_{e1} = M_u + M_t$$

$$M_t = T_u \left( \frac{1 + \frac{D}{b}}{1.7} \right)$$

$M_t = 52.94 \text{ kN.m}$

$M_{e1} = 55 + 52.94 =$

Step: 2 - Balanced BM

IS: 456-2000 - P.NO: 96 G.1.1(c)

$$M_{ud} = 0.36 \frac{x_{u,max}}{d} \left[ 1 - 0.42 \frac{x_{u,max}}{d} \right] f_{ck} b d^2$$

$\frac{x_{u,max}}{d} = 0.48$  - P.NO: 69

$M_{ud} = 139.69 \text{ kNm}$

Step: 4 - check

$M_{e1} < M_{ud}$  (Under reinforced section)

Step: 5 -  $A_{st}$

IS: 456-2000; P.NO: 96, G.1.1(b)

$$M_{e1} = 0.87 f_y A_{st} d \left[ 1 - \frac{A_{st} \cdot f_y}{b d f_{ck}} \right]$$

$A_{st} = 774.84 \text{ mm}^2$

No. of bar  $n = \frac{A_{st}}{A_{\phi}}$  Assume 20mm #

Step: 6 - ~~check for shear stress~~

IS: 456-2000 - P.NO: 75, Cl: 41.3.1

$V_e = V_u + 1.6 \frac{T_u}{b}$

$V_e = 232 \text{ kN}$

a) - Nominal shear stress ( $\tau_v$ )

$$\tau_{ve} = \frac{V_u}{bd}$$

$$= 2.06 \text{ N/mm}^2$$

b) Permissible/design shear stress ( $\tau_c$ )

P.N.O. 73, TABLE 19

$$\frac{100 A_{st}}{bd} = \dots$$

c) check for shear stress

$$\tau_{ve} > \tau_c \text{ (shear reinforcement is necessary)}$$

Step: 7 - Design of shear reinforcement

Assume 2 legged, 8 mm # vertical stirrups.

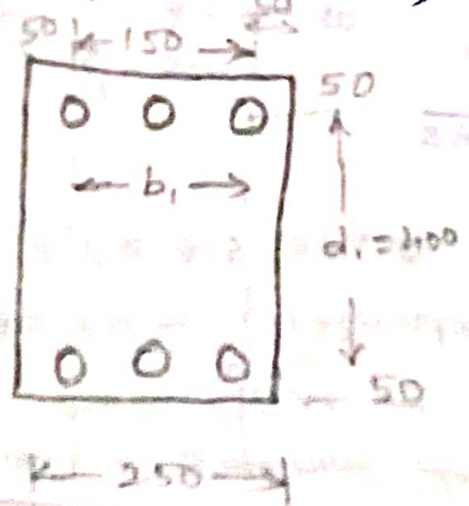
a) spacing

IS 456:2000; P.N.O. 75, 41.4.3

$$A_{sv} = \frac{T_u \cdot S_v}{b_1 d_1 (0.87 f_y)} + \frac{V_u \cdot S_v}{2.5 d_1 (0.87 f_y)}$$

$$A_{sv} = 2181 \frac{\text{mm}^2}{\text{m}}$$

$$= 100.53 \text{ mm}^2$$



Eff. cover = 35 to 50 mm

$$S_v = ?$$

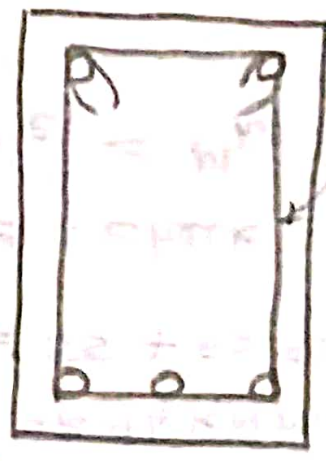
(ii) P.N.O. 75, 41.4.3

$$A_{sv} = \frac{(\tau_{ve} - \tau_c) b \cdot S_v}{0.87 f_y}$$

$$S_v = ?$$

Take least value

Step: 8 - Reinforcement details



2 legged  
8 mm #  
vertical  
stirrups @  
100 mm c/c

A reinforced concrete beam of  $\square^m$  section having a width of 300 mm & overall depth of 600 mm is reinforced with 4 bars of 25 mm dia distributed at each of the corners at an effective cover of 50 mm in the direction of width, 8 mm dia two legged stirrups are provided at 100 mm centres. Estimate the torsional strength of the section adopting Fe 415 HYSD bars for the following cases.  
(a) Torsional strength if  $V_u = 0$   
(b) Torsional strength if  $V_u = \dots$

### Given Data

$$b = 300 \text{ mm}$$

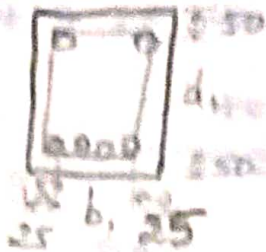
$$D = 600 \text{ mm}$$

$$b_1 = 250 \text{ mm}$$

$$d_1 = 500 \text{ mm}$$

$$S_v = 100 \text{ mm}$$

$$A_{sv} = \frac{4 \times \pi \times 8^2}{4} = 100.53 \text{ mm}^2$$



### Case: (a) - Torsional Strength ( $V_u = 0$ )

IS: 456-2000; P.No: 75, Cl: 41.4.3

$$A_{sv} = \frac{T_u \cdot S_v}{b_1 d_1 (0.87 f_y)} + \frac{V_u \cdot S_v}{2.5 d_1 (0.87 f_y)}$$

$$\therefore T_u = 45.13 \text{ kN.m}$$

### Case: (b) - Torsional Strength ( $V_u = 100 \text{ kN}$ )

IS: 456-2000; P.No: 75; Cl: 41.4.3

$$A_{sv} = \frac{T_u \cdot S_v}{b_1 d_1 (0.87 f_y)} + \frac{V_u \cdot S_v}{2.5 d_1 (0.87 f_y)}$$

$$T_u = 35.13 \text{ kN.m}$$

$\therefore$  design ultimate torsional strength = 35.13 kNm (smaller of two values)

③ A reinforced concrete beam of rectangular section with a breadth of 300 mm and overall depth 850 mm is reinforced with 4 bars of 20 mm dia on the tension side at an effective depth of 800 mm. The section is subjected to a factored B.M of 200 kN.m. If  $f_{ck} = 25 \text{ N/mm}^2$  &  $f_y = 415 \text{ N/mm}^2$ , Calculate the ultimate torsional resistance that can be allowed on the section.

### Given data

$$b = 300 \text{ mm}$$

$$d = 800 \text{ mm}$$

$$D = 850 \text{ mm}$$

$$f_{ck} = 25 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

$$A_{st} = \frac{4 \times \pi \times 20^2}{4}$$

$$= 1256 \text{ mm}^2$$

$$M_u = 200 \text{ kNm}$$

### Neutral axis depth

P.No: 70,  $x_{u \max} = 0.48$

$$x_{u \max} = 384 \text{ mm}$$

P.No: 96, Cl: 1.1.1(a)

$$\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b d}$$

$$x_u = 168 \text{ mm}$$

$$x_u < x_{u \max}$$

under reinforced

### Equivalent ultimate moment capacity of section

P.No: 96, Cl: 1.1.1(b)

$$M_e = 0.87 f_y A_{st} d \left[ 1 - \frac{A_{st} f_y}{b d f_{ck}} \right]$$

$$= 331 \text{ kN.m}$$

### Allowable Torsional Moment

Cl: P.No: 75, Cl: 41.4.3

$$M_{e1} = M_u + M_e$$

$$M_e = T_u \left[ \frac{1 + D/b}{1.7} \right]$$

$$M_{e1} = 331 \text{ kN.m}$$

$$M_u = 200 \text{ kNm}$$

$$\therefore T_u = ?$$

$$T_u = 58.1 \text{ kNm}$$

# Design of RC members for combined bending shear & Torsion

① Design the  $\square^m$  beam section for combined bending shear & torsion with the following data; size of beam =  $300 \times 600$  mm  
 shear force  $V_u = 95$  kN; Torsional moment  $T_u = 45$  kNm; BM due to external load  $115$  kNm; Use M20 & Fe415 grades.

Given data	Equivalent shear stress	Equivalent Moment	A <sub>st</sub>
b = 300 mm d = 600 mm V <sub>u</sub> = 95 kN T <sub>u</sub> = 45 kNm M <sub>u</sub> = 115 kNm f <sub>ck</sub> = 20 N/mm <sup>2</sup> f <sub>y</sub> = 415 N/mm <sup>2</sup>	$\tau_{ve} = \frac{V_e}{bd}$ P.NO: 72 (1:40.1) $V_e = V_u + 1.6 \frac{T_u}{b}$ P.NO: 75, (1:41.3) V <sub>e</sub> = 335 kN $\tau_{ve} = 1.99 \text{ N/mm}^2$	$M_e = M_u + M_t$ P.NO: 75, (1:41.4.2) $M_t = T_u \left( \frac{1 + D/b}{1.7} \right)$ M <sub>t</sub> = 79.41 kNm M <sub>e</sub> = 194.4 kNm	$\phi = 25 \text{ mm}$ clear cover = 25 SP: 16 - P.NO: 48 $M_e = \frac{194.4 \times 10^6}{bd^2}$ $bd^2 = \frac{300 \times 562.5^2}{2.05}$ P.E 2.04 → 0.655 2.06 → 0.662 ∴ 2.05 → 0.659%
No. of bars	Side reinforcement	Actual % Tensile reinf	A <sub>st1</sub>
Provide 25mm $\phi$ $n = \frac{A_{st}}{A\phi}$ n = 3 nos Provide 3-25mm $\phi$	IS: 456-2000 P.NO: $A_{st2} = 0.05\% \text{ } bD$ = 90 mm <sup>2</sup> No. of bar $\phi = 8$ mm $n = \frac{A_{st2}}{A\phi} = 1.79$ Provide - 2 - 8mm $\phi$	$P_t = \frac{100 A_{st}}{bd}$ $= \frac{100 \times 3 \times \pi \times 25^2 / 4}{300 \times 562.5}$ P <sub>t</sub> = 0.87%	$A_{st1} = P_t b D$ = 1185.3 mm <sup>2</sup> Permissible shear stress $\tau_c$ SP: 16-1980 P.NO: 178, Table: 61 0.8 → 0.57 0.9 → 0.60 0.87 → 0.57 $\left( \frac{0.6057}{0.9-0.8} \right)$ $0.87 \rightarrow 0.591 \text{ N/mm}^2$ $\tau_{ve} > \tau_c$ 1.99 > 0.59 ∴ shear reinforcement is necessary

## Design of Shear reinforcement

IS: 456-2000 P.NO: 73

$$V_{us} = V_e - \tau_c \cdot bd = 335 \times 10^3 - 0.591 \times 300 \times 562.5$$

$$V_{us} = 235.3 \times 10^3 \text{ N} = 235.3 \text{ kN}$$

## Spacing

SP: 16 - P.NO: 179; table: 62

$$\frac{V_{us}}{d} = \frac{235.3 \times 10^3}{562.5} = 418.26 \text{ N/mm}$$

SP: 16, Table: 62

- 453.7 → 80 mm
- 403.3 → 90 mm

$$\therefore 418.26 \rightarrow 80 + \left( \frac{90-80}{453.7-403.7} \right) (453.7 - 418.26)$$

Provide 2-legged 8 mm  $\phi$  stirrups @ \_\_\_\_\_ mm c/c

A T beam slab floor of an office comprises of a slab 150mm thick spanning between ribs spaced at 3m centres. The effective span of the beam is 8m. Live load on floors is 4 kN/m<sup>2</sup>. Using M20 grade concrete and Fe415 HYSD bars, design one of the intermediate beam beams.

Data

$l = 8m$ ;  $D_f = 150mm$ ;  $L.L = 4 kN/m^2$ ; spacing of T-beam = 3m  
 $f_{ck} = 20 N/mm^2$ ;  $f_y = 415 N/mm^2$ .

Cross-Sectional Dimensions

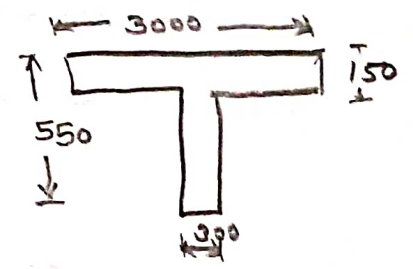
For SS beam,  $\frac{3m}{depth} = 20$

For T-beam, Assume  $b_w = 300mm$ ;  $b_f = 3m$

$\frac{b_f}{b_w} = 0.1$

Reduction factor = 0.8 ;  $\frac{basic\ span}{depth} = 20 \times 0.8 = 16$

$\therefore d = \frac{8000}{16} = 500mm$   
 $D = 550mm$



Load

slab wt (slab) =  $0.15 \times 3 \times 25 = 11.25 kN/m$

F.F.L =  $0.6 \times 3 = 1.8 kN/m$

Self wt (web) =  $0.3 \times 0.4 \times 25 = 3 kN/m$

Plaster finishes =  $0.45 kN/m$

L.L =  $4 kN/m$

Total load

$W = 20.5 kN/m$

$\therefore$  Ultimate load  $W_u = 1.5 \times 20.5 = 30.75 kN/m$

Ultimate moments & shear force

$M_u = Wl^2/8 = 246 kN.m$

$V_u = \frac{Wl}{2} = 123 kN$

Eff. width of flange

- (i)  $b_f = \frac{l_0}{6} + b_w + 6 D_f = 2530 mm$
  - (ii) c/c to rib =  $(3 - 0.3) = 2700 mm$
- Take least value  $\therefore b_f = 2530 mm$ .

Moment Capacity of flange

$M_{uf} = 0.36 f_{ck} \cdot b_f \cdot D_f (d - 0.42 D_f) = 1194 kN.m$

$M_u < M_{uf}$ ;  $x_u < D_f$  ( $\therefore$  The section is considered as rectangular width  $b = b_f$ )

Reinforcement

$M_u = 246 kN.m$   
 $d = 500 mm$   
 $b = 2530$

$M_u = 0.87 f_y \cdot A_{st} \cdot d \left[ 1 - \frac{f_y \cdot A_{st}}{f_{ck} \cdot b \cdot d} \right]$

$A_{st} = 1417 mm^2$

Assume 25mm #

No. of bars (n) =  $\frac{A_{st}}{A_\phi} = 2.88 \approx 3 nos.$  of two hanger bars (12mm)



∴ Provide 3 nos 25mm # & 2-hanger bar 12mm #

### Shear reinforcement

$b = b_w$

$$\tau_v = \frac{V_u}{b_w d} = 0.82 \text{ N/mm}^2$$

$$P_t = \frac{100 A_{st}}{b_w d} = 0.98$$

IS: 456-2000; P.No: 73; table: 19,  $\tau_c = 0.60 \text{ N/mm}^2$

$\tau_v > \tau_c$  (∴ shear reinforcement necessary)

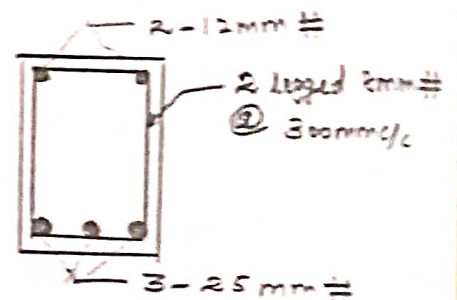
$$\text{Balance shear} = V_{us} = (V_u - \tau_c \cdot b_w \cdot d) = 33 \text{ kN}$$

Use 8mm # 2 legged stirrups

### Spacing

$$\begin{aligned} \text{(i)} \quad S_v &= \frac{0.87 f_y \cdot A_{sv} \cdot d}{V_{us}} = 547 \text{ mm} \\ \text{(ii)} \quad &\nrightarrow 0.75d = 375 \text{ mm} \\ \text{(iii)} \quad &300 \text{ mm} \end{aligned} \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} \text{Take least one} \\ \\ \therefore S_v = 300 \text{ mm} \end{array}$$

Hence provide 2 legged 8mm # stirrups @ 300mm c/c.



### November/December 2016

- ① Design a T-beam section with a flange width 1200mm, a flange depth of 100mm, a web width of 250mm and an effective depth of 500mm which is subjected to a factored moment of 550 kN.m. The concrete mix to be used is of grade M20 and steel is of grade Fe415. Use limit state method

### Given

$$\begin{aligned} b_f &= 1200 \text{ mm}; & b_w &= 250 \text{ mm}; & f_{ck} &= 20 \text{ N/mm}^2; & M_u &= 550 \text{ kNm} \\ D_f &= 100 \text{ mm} & d &= 500 \text{ mm}; & f_y &= 415 \text{ N/mm}^2; \end{aligned}$$

### Moment of resistance

Assume, N.A depth  $x_u = D_f$

$$M_{uR} = 0.36 f_{ck} b_f D_f (d - 0.42 D_f) = 395.71 \text{ kN.m}$$

$$M_{uR} < M_u$$

$$\therefore x_u > D_f$$

$$\text{So } \frac{D_f}{d} = \frac{100}{500} = 0.2 = 0.2$$

### Area of Reinforcement

$x_u$  is found, Substituting

$$M_u = 0.36 f_{ck} x_u b_f (d - 0.42 x_u)$$

$$x_u = 144.97 \text{ mm}$$

$x_{u \max}$  by  $x_u$

$$4.22 \times 10^4 x_u - 3628.8 x_u^2$$

$$x_u = 144.97 \text{ mm} > D_f = 100 \text{ mm}$$

(13)

$$\frac{D_f}{d} = 0.2 \neq \frac{D_f}{x_u} = 0.69 > 0.43$$

∴ substituting  $x_{u \max}$  by  $x_u$ .

Reinforcement

$$M_u = 0.87 f_y A_{st} E d \left[ 1 - \frac{f_y A_{st}}{f_{ck} b d} \right]$$

$$A_{st} = 3011 \text{ mm}^2$$

Assume 25 mm #

$$\text{No. of bars} = \frac{A_{st}}{A_\phi} = 6 \text{ Nos.}$$

April/May 2017

1. design the reinforcements required for a rectangular beam section with the following data.  
 Use M20 concrete + FE415 steel. Adopt limit state design method  
 size of the beam = 400 x 800 mm  
 Factored shear force = 100 kN  
 Factored tension = 50 kN ——— Torsional moment = 50 kN.m  
 Factored bending moment = 120 kN.m.

Given data

$b = 400 \text{ mm}$  ;  $D = 800 \text{ mm}$  ;  $d = 750 \text{ mm}$  ;  $f_{ck} = 20 \text{ N/mm}^2$  ;  $f_y = 415 \text{ N/mm}^2$   
 $M_u = 120 \text{ kN.m}$  ;  $T_u = 50 \text{ kN.m}$  ;  $V_u = 100 \text{ kN}$ .

Longitudinal reinforcement IS: 456:2000 ; P.No: 75 ; Cl: 41.4.2

$M_{e1} = M_u + M_{Te}$   
 $M_{e1} = 208.24 \text{ kN.m}$

$M_{Te} = T_u \left( \frac{1 + D/b}{1.7} \right) = 88.24 \text{ kN.m}$

Balanced BM IS: 456-2000 ; P.No: 96, Cl: 1.1 (c)

$M_{bd} = 0.36 \frac{x_{u \max}}{d} \left[ 1 - 0.42 \frac{x_{u \max}}{d} \right] f_{ck} b d^2$

$\frac{x_{u \max}}{d} = 0.48$

$M_{bd} = 310.42 \text{ kN.m} > M_{e1}$  (under reinforced section)

Area of reinforcement ( $A_{st}$ ) IS: 456-2000 ; P.No: 96, Cl: 1.1 (b)

$M_u = 0.87 f_y A_{st} d \left[ 1 - \frac{A_{st} f_y}{b d f_{ck}} \right]$

$b = 400 \text{ mm}$   
 $M_u = M_{e1}$

$A_{st} = 786.1 \text{ mm}^2$  , Assume 20mm #

No. of bar  $n = \frac{A_{st}}{A_{\phi}} = 2.5 \approx 3 \text{ nos.}$  ( $A_{st})_{act} = 942.5 \text{ mm}^2$

Provide 3 nos 20mm #

Check for Shear stress IS: 456-2000 ; P.No: 75 ; Cl: 41.3.1

$V_e = V_u + 1.6 \frac{T_u}{b} = 200 \text{ kN}$

IS: 456-2000  
 P.No: 72  $\rightarrow \tau_v = \frac{V_e}{b d} = 0.67 \text{ N/mm}^2$

P.No: 73  
 Table 19  $\rightarrow \tau_c = \frac{100 A_{st}}{b d} = 0.31$  ;  $\tau_c = 0.40 \text{ N/mm}^2$

$\tau_{ve} > \tau_c$  ( $\therefore$  shear reinforcement is necessary)

Shear reinforcement

Assume 2 legged 8 mm # vertical stirrups.

spacing

$$(i) A_{sv} = \frac{T_u \cdot S_v}{b_1 d_1 (0.87 f_y)} + \frac{V_u \cdot S_v}{2.5 d_1 (0.87 f_y)}$$

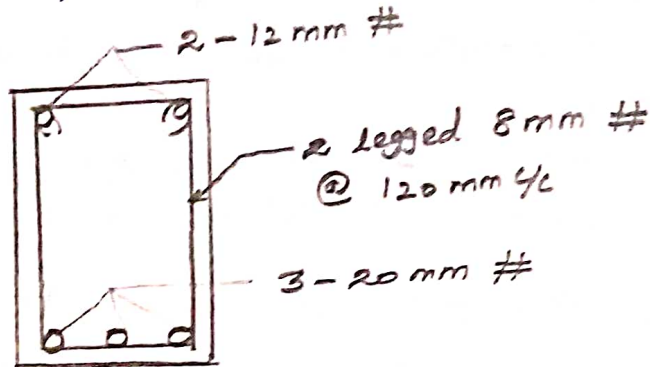
$$S_v = 122.35 \text{ mm} \approx 120 \text{ mm}$$

$$(ii) A_{sv} = \frac{(\tau_{ve} - \tau_c) b \cdot S_v}{0.87 f_y}$$

$$S_v = 336.10 \text{ mm}$$

Take least value  $\therefore S_v = 120 \text{ mm}$

Reinforcement Details

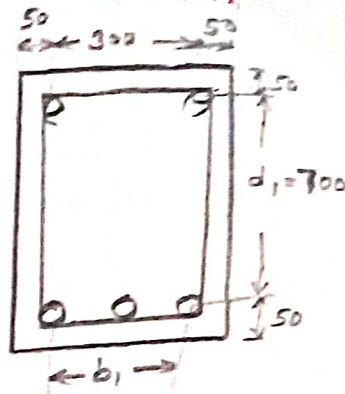


IS: 456-2000; P. 110

Cl: 41.1

$$A_{sv} = 2 \times \pi \times \left(\frac{8}{4}\right)^2 \times 120$$

$$= 100.54 \text{ mm}^2$$

IS: 456-2000; P. No: 75  
Cl: 41.4.3

2. Design a rectangular beam section of 250 mm width and 500 mm overall depth subjected to ultimate values of bending moment of 40 kN.m, shear force 40 kN, Torsion moment of 30 kN.m. Adopt eff. cover of 50 mm top & bottom. Use M20 concrete and Fe415 steel.

Same as previous problem procedure

Ans

$$M_E = T_u \left( \frac{1 + D/b}{1.7} \right) = 52.94 \text{ kN.m}$$

$$M_{e1} = M_u + M_E = 92.94 \text{ kN.m}$$

$$\text{Balanced B.M } M_{bd} = 139.69 \text{ kN.m}$$

$$M_{e1} < M_{bd} \quad (\therefore \text{under reinforced section})$$

$$M_{e1} = 0.87 f_y A_{st} d \left[ 1 - \frac{f_y A_{st}}{f_{ck} b d} \right]$$

$$A_{st} = 587.97 \text{ mm}^2$$

$$\text{No. of bars} = \frac{A_{st}}{A_{\phi}} = 3 \text{ nos.}$$

Assume 16 mm #

$$A_{st \text{ act}} = 603.19 \text{ mm}^2$$

$$V_e = V_u + 1.6 \left( \frac{T_u}{b} \right) = 232 \text{ kN}$$

$$\tau_{ve} = 2.06 \text{ N/mm}^2$$

$$P_L = \frac{100 A_{st}}{b d} = 0.54$$

$$\tau_c = 0.49 \text{ N/mm}^2$$

$\tau_{ve} > \tau_c$  (shear reinforcement is necessary)

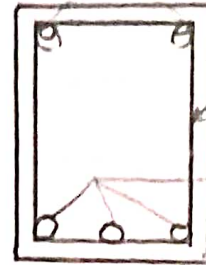
IS: 456-2000  
P. No: 96  
Cl: 41.10

shear reinforcement

$$(i) A_{sv} = \frac{T_u \cdot S_v}{b_1 d_1 (0.87 f_y)} + \frac{V_u \cdot S_v}{2.5 d_1 (0.87 f_y)} ; S_v = 67.2 \text{ mm} \approx 65 \text{ mm}$$

$$(ii) A_{sv} = \frac{(T_{ve} - T_c) \cdot b \cdot S_v}{0.87 f_y} ; S_v = 92.5 \approx 90 \text{ mm}$$

take least value,  $S_v = 65 \text{ mm}$



2-legged, 8mm # @ 65mm c/c  
 2-12mm #

Nov/Dec 2016

1 design a shear of rectangular reinforced concrete beam section to carry a factored bending moment of 220 kN.m; factored shear force of 140 kN, and a factored torsional moment of 80 kNm. Use M20 grade concrete and Fe415 steel.

Given

$$M_u = 220 \text{ kN.m} ; V_u = 140 \text{ kN} ; T_u = 80 \text{ kNm} ; f_{ck} = 20 \text{ N/mm}^2 ; f_y = 415 \text{ N/mm}^2$$

Assume c/s of beam = 300 x 850 mm. ; d = 800 mm

$$M_L = T_u \left( \frac{1 + D/b}{1.7} \right) = 172.55 \text{ kN.m}$$

$$M_{e1} = M_u + M_L = 392.55 \text{ kN.m}$$

$$\text{balanced B.M, } M_{bd} = 0.36 \frac{x_{max}}{d} \left[ 1 - 0.42 \frac{x_{max}}{d} \right] f_{ck} b d^2$$

$$M_{bd} = 529.78 \text{ kN.m}$$

$M_{bd} > M_u$  (under reinforced section)

A<sub>st</sub>

$$M_{e1} = 0.87 f_y A_{st} d \left[ 1 - \frac{f_y A_{st}}{f_{ck} b d} \right]$$

$$A_{st} = 1410.66 \text{ mm}^2 ; \text{ Assume } 25 \text{ mm } \#$$

$$\text{No. of bars } (n) = \frac{A_{st}}{A_{\phi}} = 3 \text{ nos.} , A_{st \text{ act}} = 1472.62 \text{ mm}^2$$

$$V_e = V_u + 1.6 \left( \frac{T_u}{b} \right) = 566 \text{ kN}$$

$$\tau_{ve} = \frac{V_e}{b d} = 2.35 \text{ N/mm}^2$$

$$\rho_l = \frac{100 A_{st}}{b d} = 0.61 ; \tau_c = 0.52 \text{ N/mm}^2$$

$\tau_{ve} > \tau_c$  (shear reinforcement is necessary)

shear reinforcement

$$(i) A_{sv} = \frac{T_u \cdot S_v}{b_1 d_1 (0.87 f_y)} + \frac{V_u S_v}{2.5 d_1 (0.87 f_y)} ; S_v = 60 \text{ mm}$$

$$(ii) A_{sv} = \frac{(T_{ve} - T_c) \cdot b \cdot S_v}{0.87 f_y} ; S_v = 66 \text{ mm}$$

Hence provide 2-legged 8mm # @ 0.87 f\_y

22. A simply supported beam of size 300mm x 500mm effective reinforced with 4 bars of 16mm dia HYSD steel at grade. Determine the anchorage length of the bars at the simply end if it is subjected to a uniform load of 350 kN at the centre of span wide, simply supported. The concrete mix of grade M20 is to be used. Draw the reinforcement details.

Given data:

$b = 300\text{mm}$ ;  $d = 500\text{mm}$ ;  $A_{st} = 4 \times \frac{\pi}{4} \times 16^2 = 804.25\text{mm}^2$

$V_u = 350\text{ kN}$ ;  $f_{ck} = 20\text{ N/mm}^2$ ;  $f_{yk} = 415\text{ N/mm}^2$

Anchorage length or development length,  $L_d = \frac{0.87 f_{yk} \phi}{4 \tau_{bd}}$  (or)  $\frac{\sigma_s \phi}{4 \tau_{bd}}$

$L_d = 758.19\text{ mm} \approx 758\text{ mm}$

$d' = 25 + \frac{16}{2} = 38\text{ mm}$  (clear cover = 25mm)

$d = 500\text{ mm}$

$P_t = \frac{100 A_{st}}{bd} = 0.54\%$

From SP:16, Table: 2

$\frac{M_u}{bd^2} = 1.68$

$M_u = 1.68 bd^2 = 126\text{ kNm}$

Check

$L_d < 1.3 \frac{M_1}{V} + L_0$

$753 < 1.3 \times \frac{126 \times 10^6}{350 \times 10^3} + L_0$

$L_0 = 285\text{ mm}$

$L_u = d$  (or)  $12\phi$

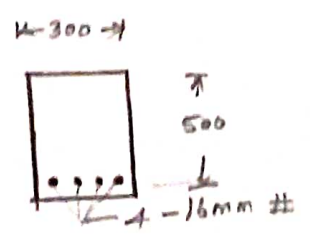
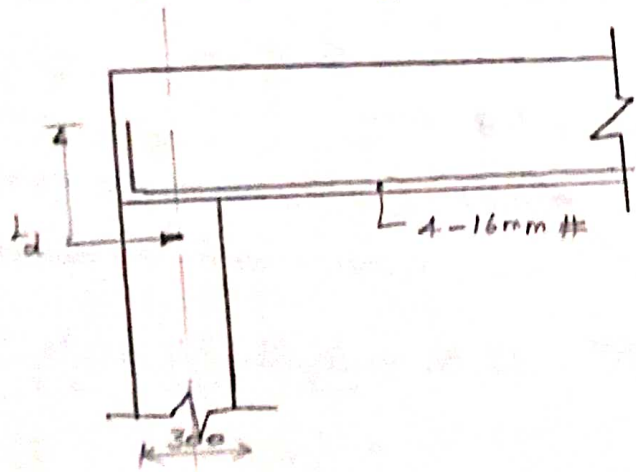
$= 500$  (or)  $12 \times 16$

$= 500$  (or)  $192\text{ mm}$

Minimum anchorage length of bar extended beyond the support

$= \frac{L_d - b}{3} = \frac{753 - 300}{3} = 151\text{ mm} < 285\text{ mm}$

Provide anchorage length  $L_d = 285\text{ mm}$ .



May/June 2016

1. Explain the terms Diagonal tension and bond stress with reference to RC beams (6-marks)

### Diagonal tension

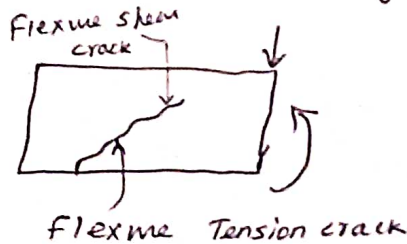
- \* Tension stresses, concern — low tensile capacity of the concrete due to horizontal bending stresses — caused bending
- \* Tension stresses various magnitude & inclinations,
  - \* shear alone (at N.A)
  - \* combined action of shear & bending
- \* Failure of <sup>RC</sup> beam. this reason — inclined tension stresses formed, it is known as diagonal tension.



little flexure cracking prior to formation of diagonal cracks

diagonal tension  $V_{cr} = \frac{V}{bd}$

- \* Large Shear, large bending



- \*  $V$  at formation of shear crack larger than web shear crack

### Bond stress

- \*  $\tau_{bd}$  → shear force per unit nominal surface area of reinforcing bar
- \* stress acting — interface b/w bars + surrounding concrete
- \*  $\tau_{bd}$  values — IS: 456-2000 ; P.No: 43, Cl: 26.2.1.1

- (b) obtain an expression for calculation of bond stress and shear stress in case of reinforced concrete beams of rectangular section with tensile steel of diameter ( $\phi$ ) Also obtain relationship between bond stress and shear stress

considered a rectangular beam

size -  $b \times d$

(Ast) Reinforcement - tension zone

Assume,  $f_{ck} = 20 \text{ N/mm}^2$  &  $f_y = 415 \text{ N/mm}^2$

### (i) Bond stress

$$U = \frac{V}{\Sigma_0 \cdot d}$$

$$U = \frac{V}{\pi D n \cdot d}$$

$V \rightarrow$  shear force

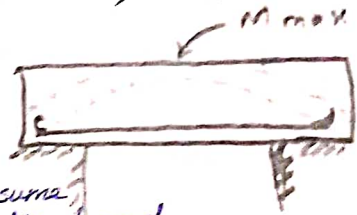
$\Sigma_0 \rightarrow$  perimeter of bar

$d \rightarrow$  eff. depth

Linear total deformation - large beam deflection, large cracks.

Bar force  $T$ ,  $U = \frac{dT}{dx}$

Bond stress  $u = \frac{U}{\Sigma_0} = \frac{V}{\Sigma_0 \cdot d}$



Assume  
No bond

\* Beam act as tied arch, will not collapse

\* Tension bar - uniform & equal

$$T = \frac{M_{max}}{Z}$$

### (ii) Shear stress

$$\tau_v = \frac{V_u}{bd}$$

$$\tau_c = \frac{100 A_{st}}{bd}$$

IS: 456: 2000 - P.No: 73



A beam of rectangular section is reinforced with 6 nos of 18 mm dia bars in tension and is supported on an eff. span of 5m, the beam being 300mm wide & 700mm deep. The beam carries a Udl of 42 kN/m. Design the shear reinforcement considering no bars are bent up for shear. Assume  $\sigma_{sv} = 230 \text{ N/mm}^2$ ;  $\tau_c = 0.3 \text{ N/mm}^2$  &  $f_y = 415 \text{ N/mm}^2$ .

Given Data

$A_{st} = 6 \times \pi \times \frac{18^2}{4} = 1526.81 \text{ mm}^2$ ; eff. span = 5m  
 $b = 300 \text{ mm}$ ;  $d = 700 \text{ mm}$ ; Udl (w) = 42 kN/m  
 $\sigma_{sv} = 230 \text{ N/mm}^2$ ;  $\tau_c = 0.3 \text{ N/mm}^2$ ;  $f_y = 415 \text{ N/mm}^2$   
 Assume  $f_{ck} = 20 \text{ N/mm}^2$ .

$\tau_v = \frac{V_u}{bd}$ ;  $V_u = \frac{W_u \cdot l}{2} = 105 \text{ kN}$ .

$\tau_v = 0.5 \text{ N/mm}^2 > \tau_c$  ( $\therefore$  shear reinforcement is necessary)

Assume 2 legged 8mm # vertical stirrups.

IS: 456-2000; P.NO: 73

$V_{us} = V_u - \tau_c \cdot bd = 37 \text{ kN}$

spacing

(i)  $V_{us} = \frac{0.87 f_y A_{sv} \cdot d}{s_v}$

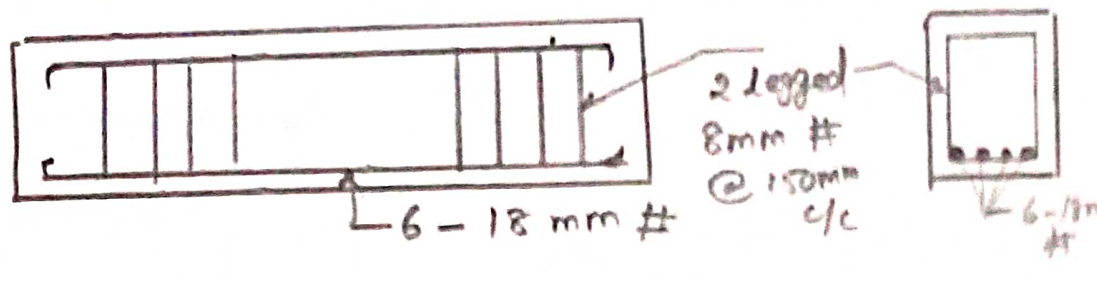
$\therefore s_v = 343 \text{ mm}$

(ii) IS: 456-2000; P.NO: 48; Cl: 26.5.1.6

$\frac{A_{sv}}{b \cdot s_v} = \frac{0.4}{0.87 f_y}$

$s_v = 151 \text{ mm} \approx 150 \text{ mm}$

Hence provide 2 legged 8mm # vertical stirrups @ 150 mm c/c



Nov/Dec 2015

- 26 1. Design a shear of rectangular reinforced concrete beam section having to carry a factored bending moment of 200 kN.m, factored shear force of 120 kN and a factored torsional moment of 75 kN.m. Use M20 concrete grade & Fe415 steel.

Same as Nov/Dec 2016 ; Problem No : 1.

changes

\* values

2. A simply supported RC beam of size 300 x 500 mm effective is reinforced with 4 bars of 16mm # HYSD steel grade Fe415. Determine the anchorage length of the bars at the simply supported end if it is subjected to a factored force 350 kN at the centre of 300 mm wide masonry support. The concrete mix of grade M20 to be used. Draw the reinforcement details.

Same as Nov/Dec 2016 ; Problem No : 2

changes

Apr/May 2015

1. A simply supported beam is 5m in span and carries a load of 75 kN/m. If 6 nos of 20mm bars are continued into the supports. Check the development length at the supports assuming M20 grade concrete & Fe415 grade steel.

Given data

$$L = 5\text{ m} ; W = 75\text{ kN/m} ; A_{st} = 6 \times \pi \times \frac{20^2}{4} = 1884.96\text{ mm}^2$$

$$f_{ck} = 20\text{ N/mm}^2 ; f_y = 415\text{ N/mm}^2 ; \phi = 20\text{ mm}$$

$$L_d \leq 1.3 \frac{M_u}{V_u} + L_0 \quad \text{For M20, } \tau_{bd} = 1.92$$

$$L_d = \frac{\phi \sigma_s}{4 \tau_{bd}} = \frac{\phi 0.87 f_y}{4 \tau_{bd}} = 940.23\text{ mm} \approx 941\text{ mm}$$

$$M_u = \frac{W_u \cdot l^2}{8} = 234.38\text{ kN.m} ; V_u = \frac{W_u \cdot l}{2} = 187.5\text{ kN}$$

$$L_0 = d \text{ (or) } 12 \phi ; d \text{ is not given}$$

$.12 \phi = 240 \text{ mm}$

$L_d \leq 1.3 \frac{M_u}{V_u} + L_o$

$941 < 1865 \text{ mm}$

Hence OK

$\therefore L_d = 941 \text{ mm}$

2. Determine the reinforcement required for a rectangular beam section with the following data.

Size of the beam =  $300 \times 500 \text{ mm}$

Factored BM =  $80 \text{ kN}\cdot\text{m}$

Factored torsional moment =  $40 \text{ kN}\cdot\text{m}$

Factored shear force =  $70 \text{ kN}$ .

Use M15 grade concrete & Fe 415 HYSD bars.

same as Apr/May 2017 ; Question no : 1.

changes

\* values.

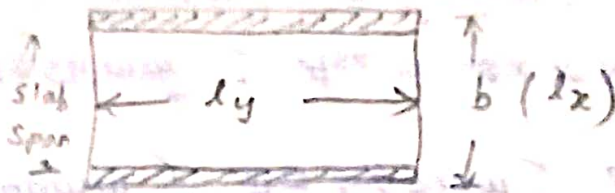
Unit - III

SLAB - ONE WAY SLAB

(Ratio of longer span to shorter span is equal to or greater than 2)

\* Main reinforcement is provided in only one direction

$$\frac{l_y}{l_x} \geq 2 \quad \text{or} \quad \frac{l}{b} \geq 2$$



Procedure

1) check (oneway/two way)

$$\frac{l_y}{l_x} \geq 2 \quad \text{(one way slab)}$$

2)

$\frac{x_{u\max}}{d}$	$f_y$	$x_{u\max}/d$
	Fe 250	0.53
	415	0.48
	500	0.46

P.NO: 70

3)  $M_u$

$$M_{u\lim} = 0.36 \frac{x_{u\max}}{d} \left[ 1 - 0.42 \frac{x_{u\max}}{d} \right] b d^2 f_{ck}$$

P.NO: 96 6.1.1(c)

4) depth of slab

$d_{pro} \quad d = \frac{\text{span}}{25} \quad \text{or} \quad d_{req} = \frac{\text{Clear (shorter) span}}{B.V \times M.F}$

Assume, Clear cover = 20mm (Assume tension reinforcement 0.8 to 1%)

$$D = d + d' + \frac{\phi}{2}$$

eff. cover = clear cover +  $\frac{\phi}{2}$

5) Eff. span P.NO: 34, clause: 22.2.(a)

- (i) clear span + eff. depth
  - (ii) c/c of supports.
- Take Least value

6) Load calculation

- self wt of slab
- Floor finish
- L.L

$b \times D \times \text{self wt of concrete}$

Total (W) Service load

Ultimate load  $W_u = 1.5 W$

P.NO: 32  
CI: 19.2.1

7) Ultimate moments & shear forces.

$$M_u = \frac{W_u l^2}{8}$$

$$V_u = \frac{W_u l}{2}$$

check for depth

$$M_u = M_{u,lim} = \frac{0.138}{1000} b d^2$$

$d = ?$   
 $d_{req} < d_{provi}$   
 Hence safe

8) <sup>check for depth of slab</sup> Limiting moment of resistance ( $M_{u,lim}$ )

$M_u = M_{u,lim}$

$$M_{u,lim} = 0.36 \frac{x_{u,max}}{d} \left[ 1 - 0.42 \frac{x_{u,max}}{d} \right] f_{ck} b d^2$$

$d_{req}$

P.No: 96 G.I.I.C

9) check

$d_{req} < d_{pro}$  Hence safe.

$$M_u < M_{u,lim} \text{ (under reinforced)}$$

10) Main reinforcement ( $A_{st}$ )

$$M_u = 0.87 f_y A_{st} d \left[ 1 - \frac{A_{st} f_y}{b d f_{ck}} \right]$$

P.No: 96 G.I.I.(b)

$$A_{st} = ?$$

Spacing Assume  $\phi$  mm

$$S = \frac{1000 A_{st}}{A_{st}}$$

$$S = \frac{1000 A \phi}{A_{st}}$$

1 bar area

$$A_{st} = \frac{\pi d^2}{4}$$

bar dia

check for spacing  
 P.No: 46 (13d)  
 $c1 = 26.3.3$   
 $b$

ii) Distribution reinforcement/secondary reinforcement

$$A_{st} = 0.12\% \text{ c/s Area}$$

$$= \frac{0.12}{100} b D$$

$$S = \frac{1000 A \phi}{A_{st}}$$

P.No: 48

Fe 250

$$A_{st, min} = 1.5 D$$

Fe 415 & Fe 500

$$A_{st, min} = 1.2 D$$

$$A_{st, min} < A_{st} \text{ Hence ok}$$

secondary check for spacing

- (i)  $5d$
  - (ii)  $+50$
  - (iii)  $\frac{1000 A\phi}{A_{st}}$
- } Take least Value P.NO: 46

Main spacing

- (i)  $3d$
  - (ii)  $300$
  - (iii)  $\frac{100 A\phi}{A_{st}}$
- } Take least Value P.No: 46

12) check for shear

P.NO: 72  $\tau_v = \frac{V_u}{bd}$   $V_u = \frac{W_u \cdot l}{2}$

P.NO: 73 Table: 19  $\tau_c \propto A_{st} = \frac{100 A_{st}}{bd} = ?$

$\tau_c = k$  — from Tabulation  
 ↑ P.NO: 72 40.2.1.1

$\tau_v < \tau_c$  Hence safe (shear stress within the permissible limit)

13) check for deflection

P.NO: 37; 23.2.1  $\frac{l}{d} = 20$

①  $\left(\frac{l}{d}\right)_{max} = \left(\frac{l}{d}\right)_{basic} \cdot k_L \cdot k_C \cdot k_F$

P.NO: 39 Fig (4)  $f = 0.58 f_y$   $k_L = \frac{P_L}{f} = \frac{100 A_{st}}{bd}$

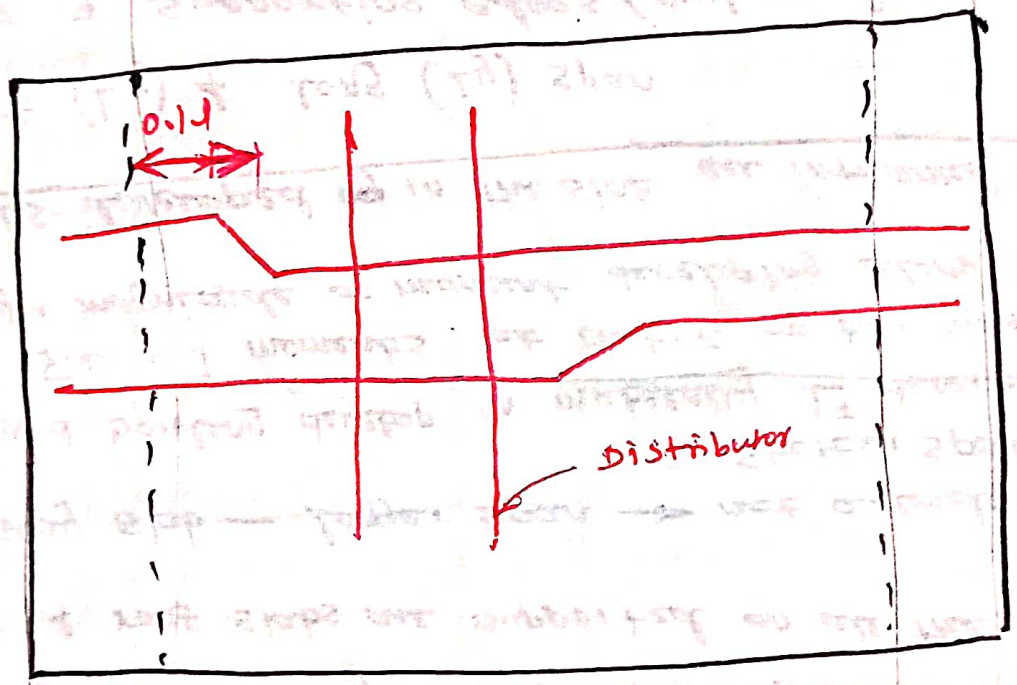
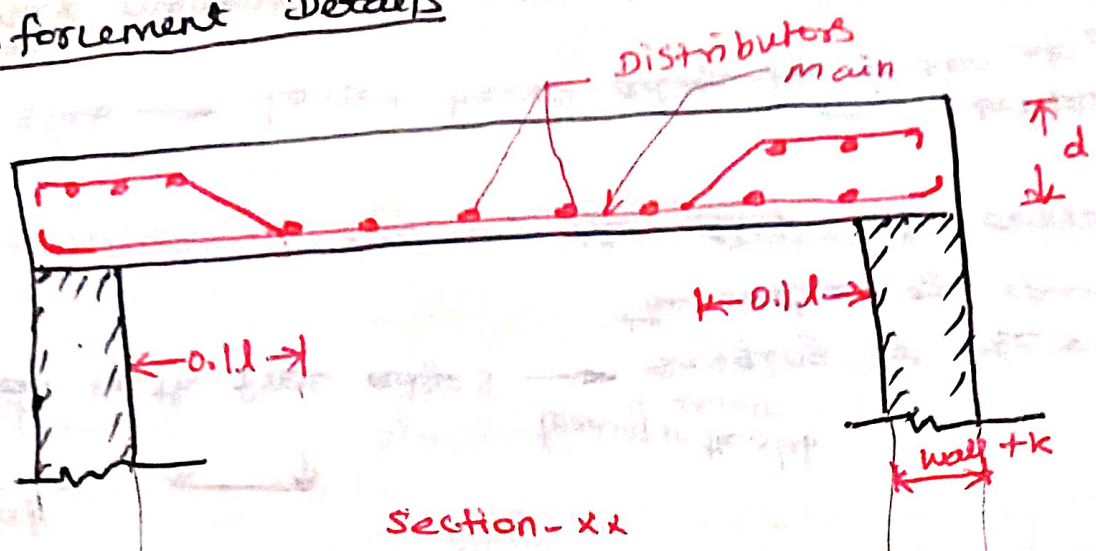
$k_C =$  Fig (5) P.NO: 39

$k_F =$  P.NO: 6 Fig: 6

②  $\left(\frac{l}{d}\right)_{act} = \frac{Eff. Span}{Eff. depth}$

Check  $\left(\frac{l}{d}\right)_{act} < \left(\frac{l}{d}\right)_{max}$  Hence safe.

(14) Reinforcement Details



Problem :-

design a one way slab with a clear span of 3.5 m simply supported on 200 mm thick concrete masonry walls to support a live load of  $4 \text{ kN/m}^2$ . Adopt M-20 grade concrete & Fe 415 HYSD bars.

Ans  
 eff. span  $L = 3.64 \text{ m}$   
 $d = 140 \text{ mm}$ ;  $d' = 20$ ;  $\phi = 10$ ;  $D = 165 \text{ mm}$   
 $W = 9.125 \text{ kN/m}$ ;  $W_u = 13.69 \text{ kN/m}$ ;  $M_u = 22.67 \text{ kNm}$ ;  $V_u = 24.92 \text{ kN}$   
 $M_{u \text{ lim}} = 54 \text{ kNm}$ ;  $M_u < M_{u \text{ lim}}$  (under); Main reinforcement  $A_{st} = 480$   
 10mm  $\phi$  spacing 164  $\approx$  160 mm; Secondary reinforcement  $A_{st} = 198$ ; 8mm  $\phi$  spacing = 250

$\tau_v = 0.178 \text{ N/mm}^2$ ;  $P_L = 0.17$ ;  $k \cdot \tau_c = 0.35 > \tau_v$  Hence shear stresses are within safe permissible limits

$\rho_L = 0.34$

$\left(\frac{L}{d}\right)_{\max} = \frac{k_L = 1.4}{k_c = 1} = 20 \times 1.4 \times 1 = 28$

$\left(\frac{L}{d}\right)_{\text{act}} = \frac{3640}{140} = 26 < 28$  Hence OK

2) Design a RCC slab of an office floor/room 3m x 7m, the thickness of supporting walls 300mm, Live load  $2 \text{ kN/m}^2$ ; Floor finish load  $1.5 \text{ kN/m}^2$ , use M25 & Fe 415 grade.

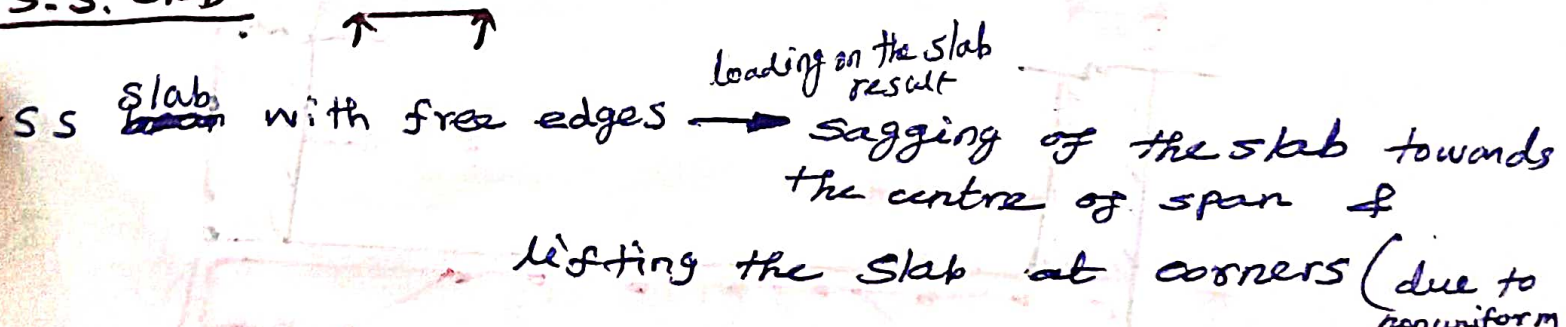
Two way slab

- \* Multi storied buildings with column & beam construction
- \* Floor & roof slabs are supported on all the four sides.
- \* Two way slab — longer span  $\rightarrow$  not exceeding 2 times of shorter span.
- (flexural bending develop in mutually  $\perp^r$  directions) reinforcement design
- max. flexural moments at centre of the slab
- Longer magnitude of moment developing along shorter span

\* Moments developed in the slab are influenced by

- Short ( $L_x$ ) & long ( $L_y$ ) span
- Type of supporting edges (such as free, fixed & continuous)
- Magnitude & type of load on the slab (point load, Udl etc)

S.S. Slab



S.S slab  $\rightarrow$  do not have adequate torsion at corners.

design  
Max. moments per unit width



$$M_x = \alpha_y \cdot W \cdot L_x^2$$

$$M_y = \alpha_x \cdot W \cdot L_x^2$$

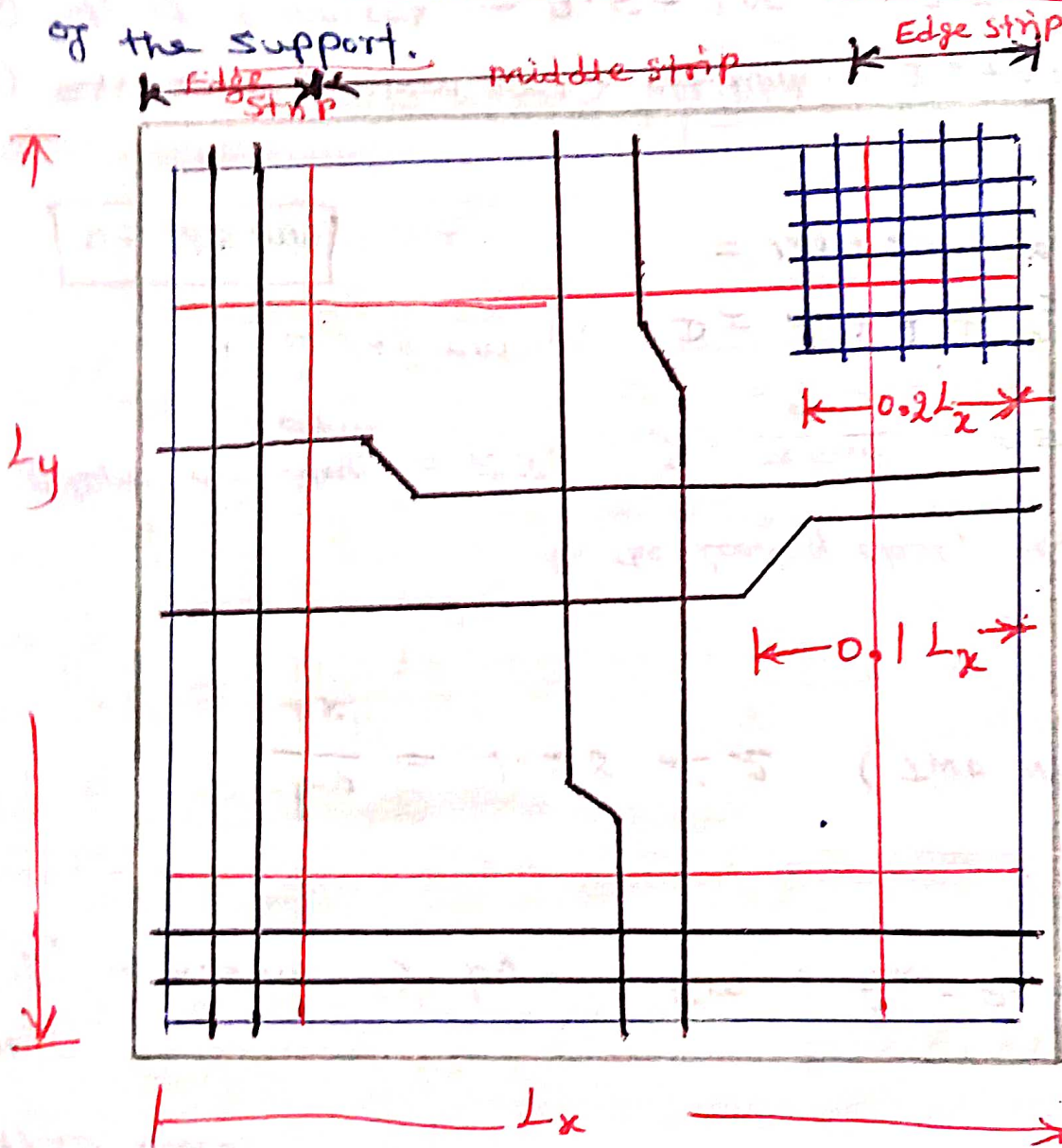
IS: 456:2000 - P.No: 91

Moment co-efficient  $\alpha_x, \alpha_y$

IS: 456:2000 - P.No: 91

Table: 27

- Moment co-efficient <sup>evaluated</sup> based on Rankine-Groshoff theory
- IS: 456:2000 - at least 50% tension reinforcement provided at midspan extend to supports.
- Remaining 50% extend to within  $0.1 L_x$  or  $0.1 L_y$  of the support.



Edge strip

$$= L_x/8$$

$$(or) \frac{L_y}{8}$$

Edge strip

$$\frac{L_y}{8}$$

Layer span

P.No: 39 - span/depth ratio - Two way slab

Design a two way slab for an office floor of size 3.5 m x 4.5 m discontinuous and simply supported edge on all the sides with corners prevented from lifting and supporting a service live load of 4 kN/m<sup>2</sup>. Adopt M-20 grade concrete and Fe415 HYSD bars.

Step: 1 - Given data

$$L_x = 3.5 \text{ m} ; L_y = 4.5 \text{ m} ; f_{ck} = 20 \text{ N/mm}^2 ; f_y = 415$$

Step: 2 - Check for one way (or) two way slab

$$\frac{L_y}{L_x} = 1.28 < 2 \quad (\text{Two way slab})$$

With provision for torsion at corners

Step: 3 - depth of slab

IS: 456: 2000

P.N.O: 39

As the leading class, the value of 3kN/m<sup>2</sup>

adopt a  $\frac{\text{span}}{\text{depth}} = 25$

$$\therefore \frac{3500}{d} = 25$$

$$\therefore d = 140 \text{ mm}$$

$$D = d + d' + \frac{\phi}{2}$$

$$= 140 + 20 + \frac{10}{2} = 165 \text{ mm}$$

over all depth  
=  $\frac{L_x}{3 \times \text{M.F}}$

$$D = 165 \text{ mm}$$

Step: 4 - Eff. span

(i) Eff. span = clean span + eff. depth = 3.5 + 0.14 = 3.64 m

(ii) c/c of supports = 3.5 +  $\frac{200}{1000}$  = 3.7 m

Assume wall thick

Take lesser value

$$l_{\text{eff}} = 3.64 \text{ m}$$

Step: 5 - load calculation:

Self wt of slab =  $b D \gamma_{\text{con}} = 1 \times 0.165 \times 25 = 4.125 \text{ kN/m}^2$

Live load =  $b \times \text{L.L} = 1 \times 4 = 4 \text{ kN/m}^2$

Floor Finish = Assume 0.6 kN/m<sup>2</sup> x b = 0.6 kN/m

Total service load = 8.73 kN/m

$\therefore$  Design ultimate load  $W_u = 1.5 \times W$

$$W_u = 13.08 \text{ kN/m}$$

Step: 6 - Ultimate Design Moment & Shear force

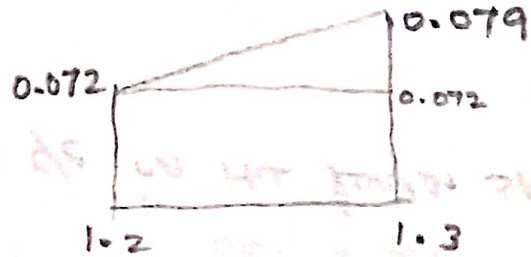
IS: 456:2000 - P. NO: 91; Table 26

Long span co-efficient  $\alpha_y = 0.056$  (From edge discontinuous)

Short span  $\alpha_x$

1.2  $\rightarrow$  0.072

1.3  $\rightarrow$  0.079



$\therefore 1.28 \rightarrow 0.072 + \left( \frac{0.079 - 0.072}{1.3 - 1.2} \right) \times (1.28 - 1.2)$

$$\alpha_x = 0.077$$

P. NO: 91

$$\begin{cases} M_{ux} = \alpha_x \cdot W_u \cdot L_x^2 = 0.077 \times 13.08 \times 3.64^2 = \underline{13.34 \text{ kNm}} \\ M_{uy} = \alpha_y \cdot W_u \cdot L_x^2 = 0.056 \times 13.08 \times 3.64^2 = \underline{9.70 \text{ kNm}} \end{cases}$$

$$V_{ux} = \frac{W_u \cdot L_x}{2} = \frac{13.08 \times 3.64}{2} = \underline{23.8 \text{ kN}}$$

Step: 7 - check for eff. depth

IS: 456-2000; P. NO: 96; Cl. 1.1.(c)

$$M_{u \text{ lim}} = 0.36 \frac{x_{u \text{ max}}}{d} \left[ 1 - 0.42 \frac{x_{u \text{ max}}}{d} \right] f_{ck} b d^2$$

13.34

$$d = 69.52 \text{ mm} < 140 \text{ mm} \quad \text{Hence ok.}$$

( $\therefore$  To resist the ultimate moment)

Step: 8 - Reinforcement (Short & longer direction)

(a) shorter direction

IS: 456:2000; P. NO: 96; Cl. 1.1.(b)

$$M_u = 0.87 f_y A_{st} d \left[ 1 - \frac{A_{st} \cdot f_y}{f_{ck} b d} \right]$$

Max = 13.34

$$A_{st} = 980 \text{ mm}^2$$

$$A_{st \text{ min}} = 0.12\% b D$$

Spacing

$$s = \frac{1000 A \phi}{A_{st}} \quad \phi = 10 \text{ mm}$$

Check for spacing

P. NO: 46

- (i) 3d
  - (ii) 300
  - (iii)  $\frac{1000 A \phi}{A_{st}}$
- } Take lesser value

Provide 10mm  $\phi$  @ 250 mm c/c.

b) Longer direction

$A_{st y}$

IS: 456:2000 - P. NO: 96. 4.1.1.(b)

$$M_u = 0.87 f_y A_{st} d \left[ 1 - \frac{A_{st} f_y}{f_{ck} b d} \right]$$

$$A_{st} = 213 \text{ mm}^2$$

Check for spacing

- (i) 3d
  - (ii) 300
  - (iii)  $\frac{1000 A \phi}{A_{st y}}$
- } Take lesser value

Provide 10mm  $\phi$  @ 300mm c/c in the longer span direction.

Step 9 - Check for shear stress

Considering shorter span direction

IS: 456-2000 - P. NO: 72

$$V_u = \frac{W_u \cdot L_x}{2}$$

$$\tau_v = \frac{V_u}{b d} = \frac{23.8 \times 10^3}{1000 \times 140} = 0.17 \text{ N/mm}^2$$

$$P_t = \frac{100 A_{st}}{bd} = 0.225$$

IS: 456:2000 - P. NO: 72 Table

$$k \cdot \tau_c = 1.27 \times 0.31 = 0.39 \text{ N/mm}^2$$

$$k \cdot \tau_c > \tau_v \quad (\text{shear within the Permissible limit})$$

Step: 10 - Check for deflection

P. NO: 37 ; Cl: 23.2.1

$$\left(\frac{L}{d}\right)_{\text{basic}} = 20$$

$$\begin{aligned} \text{(ii)} \left(\frac{L}{d}\right)_{\text{max}} &= B \cdot V \times K_t \\ &= 20 \times 1.6 \\ &= 32 \end{aligned}$$

$$P_t = 0.225 ; K_t = 1.6$$

$$\left(\frac{L}{d}\right)_{\text{act}} = \frac{L_x}{d} = \frac{3640}{140} = 26 < 32$$

$$\left(\frac{L}{d}\right)_{\text{act}} < \left(\frac{L}{d}\right)_{\text{max}} \quad (\text{Hence safe})$$

Step: 11 - Check for Crack Control

$$\text{(i)} A_{st \text{ provi}} > 0.12 \% bD$$

$$\text{(ii)} \text{Spacing of main reinforcement} \neq 3 \cdot d$$

$$\text{(iii)} \text{dia of reinforcement} < \left(\frac{D}{8}\right) < \left(\frac{165}{8}\right)$$

Hence, Crack will be within ~~the~~ safe permissible limit

Step: 12 - Torsion reinforcement at corners

**NOTE**  
Corners held down  
Torsion is necessary

$$\left( A_{st \text{ of torsion @ each corners}} = \frac{3}{4} A_{st} = 236 \text{ mm}^2 \right)$$

$$\text{Length} = \frac{1}{5} \text{ Shorter span} = \frac{1}{5} \times 3500 = 700 \text{ mm}$$

Assume  $8 \text{ mm } \phi$

Spacing, 4 corners, 4 layers.

Edge strips

$$A_{st} = 0.12\% bD = 198 \text{ mm}^2$$

Provide  $10 \text{ mm } \phi @ 300 \text{ mm } \phi$ .

$$A_{st} = 262$$

### Cantilever beam

\* minimum distribution reinforcement provided in transverse direction

\* check - deflection, cracking & failure due to overturning

$$* \frac{\text{Span}}{\text{Depth}} = 7$$

\* provide anchorage length near support to main reinforcement (avoid failure due to anchorage)

Design a cantilever chajja (Balcony) slab projecting  $1 \text{ m}$  from the support using  $M20$  grade concrete &  $Fe415$  HYSD Bars. Adopt a Live load of  $3 \text{ kN/m}^2$

Step: 1 - Given data

$$L = 1 \text{ m} ; \quad LL (q) = 3 \text{ kN/m}^2 ; \quad f_{ck} = 20 \text{ N/mm}^2 ; \quad f_y = 415$$

Step: 2 - Depth of slab

$$\frac{\text{Span}}{\text{Depth}} = 7 \quad \text{IS: 456: 2000 - P.No: 37 ; Cl: 23.2.1}$$

$$d = 142.8 \text{ mm} \quad \text{say } d = 150 \text{ mm} \quad (a)$$

$$D = d + d' + \frac{\phi}{2} = 150 + 20 + \frac{10}{2}$$

$$D = 175 \text{ mm}$$

Adopt maximum depth of  $150 \text{ mm}$  at support - gradually reducing to  $100 \text{ mm}$  at the free end.

Step: 3 - Load calculation

$$\text{Self wt of slab} = \left( \frac{0.15 + 0.1}{2} \right) \times 1 \times 25 = 3.125 \text{ kN/m}$$

$$\text{Live load} = 3.00 \times 1 = 3 \text{ kN/m}$$

$$\text{Floor finish load} = 0.875 \times 1 = 0.875 \text{ kN/m}$$

∴ Total working load =  $\frac{7.00 \text{ kN/m}}{1}$

Design ultimate load ( $W_u$ ) =  $1.5 \times 7 = 10.5 \text{ kN/m}$

Step: 4 - Ultimate moments & Shear forces.

$$M_u = \frac{W_u \cdot l^2}{2} = \frac{10.5 \times 1^2}{2} = 5.25 \text{ kN}$$

$$V_u = W_u \cdot l = 10.5 \times 1 = 10.5 \text{ kN}$$

Step: 5 - Check for depth

$$M_{u \text{ lim}} = 0.36 \frac{x_{u \text{ max}}}{d} \left[ 1 - 0.42 \frac{x_{u \text{ max}}}{d} \right] f_{ck} b d^2$$

$d = ? < d_{\text{prov}}$  Hence safe (to resist ultimate moment)

25:456:2000  
P.NO: 96

Step: 6 -  $A_{st}$

$$M_u = 0.87 f_y \cdot A_{st} \cdot d \left[ 1 - \frac{A_{st} \cdot f_y}{b d f_{ck}} \right]$$

P.NO: 96

G.I.(b)

$$A_{st} = ?$$

10 mm dia

$A_{st \text{ min}}$

$$Fe \ 250 \rightarrow A_{st \text{ min}} = 0.15\% \cdot b D$$

$$Fe \ 415 \rightarrow A_{st \text{ min}} = 0.12\% \cdot b D$$

$A_{st} > A_{st \text{ min}}$  Hence OK.

otherwise provide just above

Distribution reinforcement, spacing same as  $A_{st \text{ min}}$  previous

Step: 7 - Anchorage length

$$L_d = \frac{0.87 f_y \phi}{4 \tau_{bd}} \quad \text{--- P.NO: 42; 26.2}$$

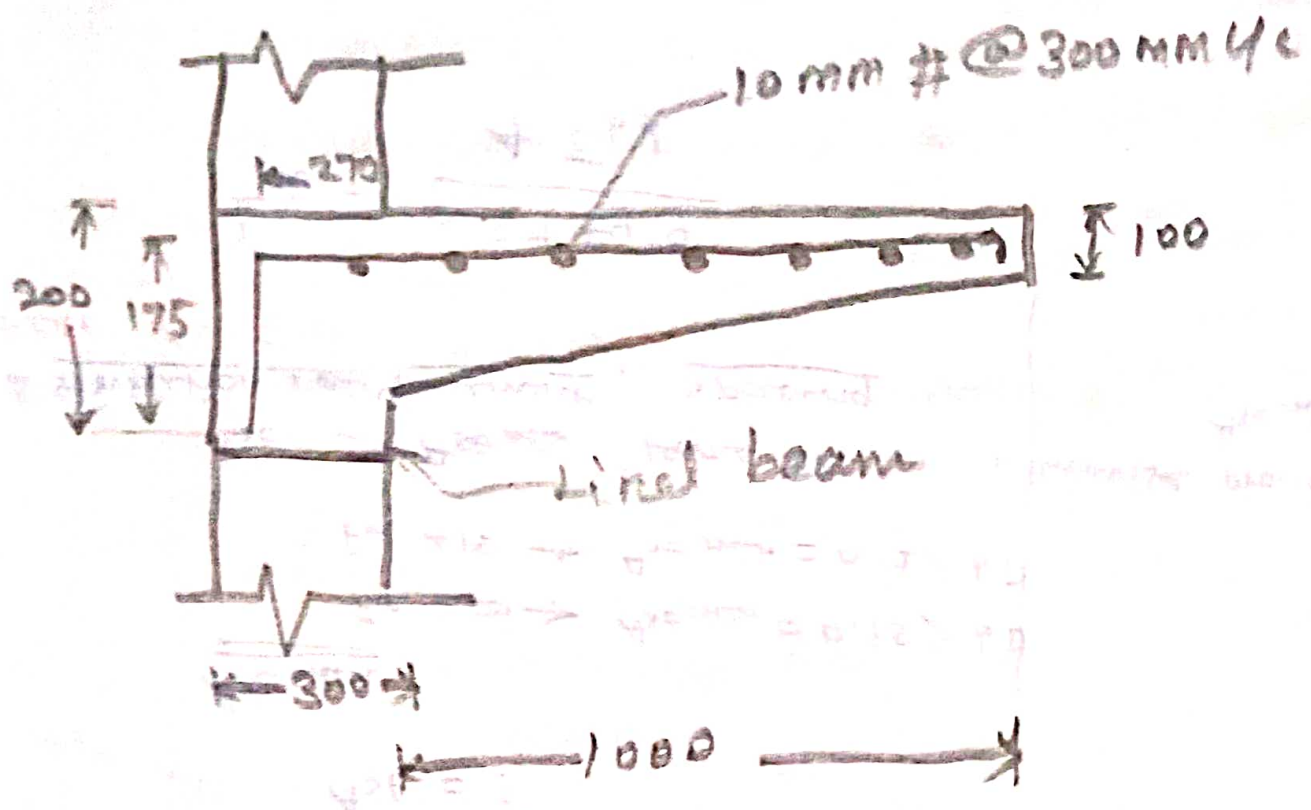
$$\text{--- P.NO: 43 - 26.2.1.1}$$

$$L_d = 470 \text{ mm}$$

Step: 8 - check for deflection

$$P.NO: 37 ; \text{Cl. 23.2.1}$$

$$\left( \frac{l}{d} \right)_{\text{Basic}} =$$



Design a slab for a Room of clear internal dimensions 3m x 5m supported on walls of 300mm thickness with corners held down. Two adjacent edges of the slab are continuous & other two are discontinuous. The Live load on the slab is  $3 \text{ kN/m}^2$ . Assume floor finish load as  $1 \text{ kN/m}^2$ . Use M20 concrete & Fe 415 Steel. Adopt limit state design method NOV/DEC 2010

A reinforced concrete slab has an effective span of 5m, and carries an udl of  $6 \text{ kN/m}^2$  inclusive of its own weight. Determine (i) eff. depth of slab (ii) steel reinforcement. Use M20 concrete & Fe 415 steel

Given data	Load calculation	Moment	eff. depth	reinf. reinforcement
$l_{\text{eff}} = 5 \text{ m}$ $u.d.l = 6 \text{ kN/m}^2$	$W_u = 6 \times 1.5$ $= 9 \text{ kN/m}^2$	$M_u = \frac{W_u l^2}{8}$	$M_u = M_{u \text{ lim}}$ $M_{u \text{ lim}} = 0.36 f_{ck} b d^2$	$m_u = 0.87 f_y A_{st}$

NOV/DEC 13



Apr/May 2017

1. design a two way slab for the following data.

size = 7m x 5m ; width of the support = 300mm  
Edge condition = two short edges are discontinuous,  
Live load = 5 kN/m<sup>2</sup> ; Floor finish = 1 kN/m<sup>2</sup> ;  
Use M20 concrete and Fe415 steel.

Given data :-

$$L_x = 5m ; L_y = 7m ; f_{ck} = 20 \text{ N/mm}^2 ; f_y = 415 \text{ N/mm}^2$$

Check  $\frac{L_y}{L_x} = \frac{7}{5} = 1.4 < 2$  ( $\therefore$  Two way slab)

Depth of slab IS: 456:2000; P.No: 39

$$\frac{\text{span}}{\text{depth}} = 25 ; \frac{5000}{d} = 25 ; d = 200 \text{ mm}$$
$$D = d + d' + \frac{\phi}{2} \quad D = 225 \text{ mm}$$

Eff. span

(i) clear span + eff. depth = 5.20 m  
(ii) % of support = 5.30 m } Take least value  
 $\therefore l_{\text{eff}} = 5.20 \text{ m}$

Load calculation :-

$$\text{self wt} = b D \gamma_{\text{con}} = 1 \times 0.225 \times 25 = 5.625 \text{ kN/m}$$

$$\text{Live load} = 1 \times 5 = 5.0 \text{ kN/m}$$

$$\text{Floor finish} = 1 \times 1 = 1.0 \text{ kN/m}$$

$$\therefore \text{Total}_{\text{service}} \text{ load} = 11.625 \text{ kN/m}$$

$$(W_u) \text{ Ultimate load} = 1.5 \times W = 17.44 \text{ kN/m}$$

Ultimate Design Moment & shear forces

$$\text{Long span co-efficient } \alpha_y = 0.035$$

$$\text{Short span co-efficient } \alpha_x = 0.044$$

$$L_x = 5.20 \text{ m}$$

IS: 456:2000 P.No: 91

$$\left. \begin{array}{l} \text{Moment} \\ M_{ux} = \alpha_x \cdot W_u \cdot L_x^2 = 20.75 \text{ kN}\cdot\text{m} \\ M_{uy} = \alpha_y \cdot W_u \cdot L_x^2 = 16.51 \text{ kN}\cdot\text{m} \end{array} \right\}$$

Shear

$$V_{ux} = \frac{W_u \cdot L_x}{2} = 45.34 \text{ kN}$$

Check for eff. depth :-

IS: 456:2000 - P.No: 96; Cl. 1.1.(c)

$$M_{u \text{ lim}} = 0.36 \frac{x_{u \text{ max}}}{d} \left[ 1 - 0.42 \frac{x_{u \text{ max}}}{d} \right] f_{ck} b d^2$$
$$d = 86.67 \text{ mm} < 200 \text{ mm} \quad \text{Here OK}$$

Reinforcement

shorter direction

IS: 456-2000; P.No: 96; 41.1.(b)

$$M_u = 0.87 f_y A_{st} d \left[ 1 - \frac{A_{st} \cdot f_y}{f_{ck} \cdot b d} \right]$$

$$A_{st} = 229 \text{ mm}^2$$

$$A_{st \text{ min}} = 0.12\% \cdot b D \\ = 240 \text{ mm}^2$$

$$\therefore A_{st} = 240 \text{ mm}^2$$

Assume 10mm  $\phi$ Spacing

least value

$$\left. \begin{array}{l} \text{(i)} 3d = 600 \text{ mm} \\ \text{(ii)} 300 = \\ \text{(iii)} \frac{1000 A \phi}{A_{st}} = \frac{1000 \times \pi \times 10^2 / 4}{240} \end{array} \right\}$$

Provide 10mm  $\#$  @ 300mm  $\phi$ .

longer direction

IS: 456: 2000 - P.No: 96; 41.1.

$$M_u = 0.87 f_y A_{st} d \left[ 1 - \frac{A_{st} \cdot f_y}{f_{ck} b d} \right]$$

$$A_{st} = 287.4 \text{ mm}^2$$

Assume, 10mm  $\#$ SpacingProvide 10mm  $\#$  @ 300mm  $\phi$ check for shear

IS: 456: 2000; P.No: 72

short span direction,

$$\tau_v = \frac{V_u}{b d} = 0.23 \text{ N/mm}^2$$

IS: 456: 2000

P.No: 72

cl: 40.2.1.1

 $\tau_c$ 

$$P_L = \frac{100 A_{st}}{b d} = 0.12$$

$$\tau_c = 0.28 \text{ N/mm}^2$$

$$k \cdot \tau_c = 1.2 \times 0.28 = 0.34 \text{ N/mm}^2$$

$$\tau_v < \tau_c \text{ (shear within the permissible limit)}$$

check for deflection

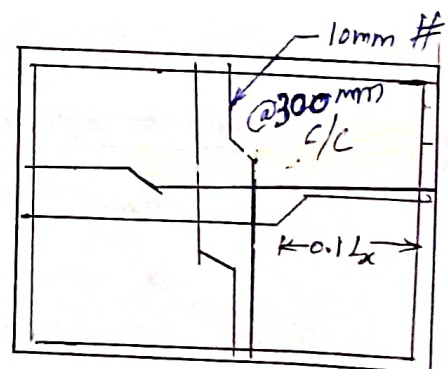
IS: 456-2000; P.No: 57; cl: 23.2.1

$$\text{(i)} \left( \frac{L}{d} \right)_{\text{basic}} = 20$$

$$\left( \frac{L}{d} \right) = \frac{5200}{200} = 26$$

$$\text{(ii)} \left( \frac{L}{d} \right)_{\text{max}} = B.V.K.z \\ = 20 \times 1.6 \\ = 32$$

$$\left( \frac{L}{d} \right)_{\text{act}} < \left( \frac{L}{d} \right)_{\text{max}} \text{ Hence safe.}$$

A T  
thick  
span  
using

2. Design a slab over a room 5m x 7m as per IS code. The slab is supported on masonry walls all around with adequate restraint and the corners held down. The live load on the slab is  $330 \text{ N/m}^2$ . The slab has a bearing of 150mm on the supporting walls.

Given data

$L_x = 5 \text{ m}; L_y = 7 \text{ m}; f_{ck} = 20 \text{ N/mm}^2$  (Assume);  $f_y = 415 \text{ N/mm}^2$  (Assume); L.L =  $330 \text{ N/m}^2 = 0.33 \text{ kN/m}^2$

check  $L_y/L_x = 1.4$  (Two way slab)

Depth of slab  $\frac{L}{d} = 25$ ;  $\frac{5000}{d} = 25$ ;  $d = 200 \text{ mm}$ ;  $D = 225 \text{ mm}$   
Assume 10mm # bar

Eff. depth (d) = 200mm; Eff. span = 5.2m.

Load calculation:

Self wt slab = $b D \rho_c$	= 5.625 kN/m	} Total load (W) = 6.96 kN/m
L.L =	= 0.33 kN/m	
FFL =	= 1 kN/m	

Bending Moment:

B.M. co-efficient  $\alpha_x = 0.099$ ;  $\alpha_y = 0.051$   
(IS 456:2000; P.No: 91, Table: 27)

Short span Moment  $M_{ux} = \alpha_x \cdot W_u \cdot l_x^2 = 27.92 \text{ kN.m}$

Long span Moment  $M_{uy} = \alpha_y \cdot W_u \cdot l_y^2 = 14.38 \text{ kN.m}$

Reinforcement

Shorter span

$M_{ux} = 0.87 f_y A_{st} d \left[ 1 - \frac{A_{st} f_y}{b d f_{ck}} \right]$

$A_{st} = 403.5 \text{ mm}^2$

Assume, 10mm #

$n = \frac{A_{st}}{A\phi} = \frac{403.5}{\pi \times 10^2 / 4} = 5.13 \text{ nos/metre}$

Spacing

$s = \frac{A\phi}{A_{st}} \times 1000 = 194.65 \text{ mm}$

Provide 10mm # @ 180mm c/c

Longer span

$M_{uy} = 0.87 f_y A_{st} d \left[ 1 - \frac{A_{st} f_y}{b d f_{ck}} \right]$

$A_{st} = 203.4 \text{ mm}^2$

$A_{st \text{ min}} = 0.12\% b D = 270 \text{ mm}^2$

spacing  $s = \frac{1000 A\phi}{A_{st}} = 292.5 \text{ mm}$

Provide 10mm # @ 250mm c/c

14

Check for shear  
considering short span

$$\tau_v = \frac{V_u}{bd} = 0.14 \text{ N/mm}^2$$

$$P_t = \frac{100 A_{st}}{bd} = 0.20$$

IS: 456-2000, PND: 73  
Table: 19

$$\tau_c = 0.32 \text{ N/mm}^2$$

$$k \cdot \tau_c = 1.2 \times 0.32 = 0.38 \text{ N/mm}^2$$

$k \cdot \tau_c > \tau_v$  ( $\therefore$  shear within the permissible limit)

Check for deflection:

$$\left(\frac{l}{d}\right)_{\text{basic}} = 20$$

$$(i) \left(\frac{l}{d}\right)_{\text{act}} = 26$$

$$(ii) \left(\frac{l}{d}\right)_{\text{max}} = 8 \cdot v \cdot k_t = 32$$

$\left(\frac{l}{d}\right)_{\text{act}} < \left(\frac{l}{d}\right)_{\text{max}}$  (Hence safe)

Torsion reinforcement at corners

$$A_{st} \text{ of torsion @ each corner} = \frac{3}{4} A_{st} = 303 \text{ mm}^2$$

(4 corner)  
4 layer

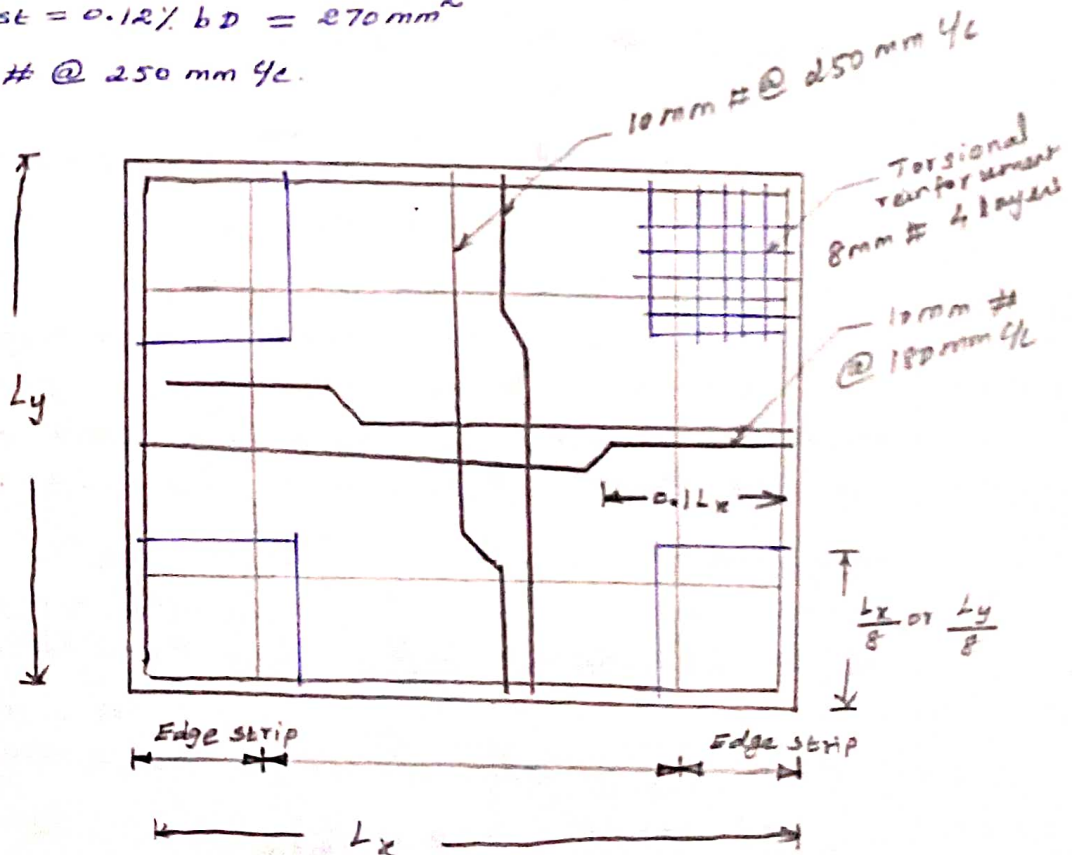
$$\text{Torsional steel length} = \frac{1}{5} \text{ shorter span} = 1 \text{ m}$$

Assume 8mm #

Edge strip

$$A_{st} = 0.12\% \text{ } bD = 270 \text{ mm}^2$$

Provide 10mm # @ 250mm  $\phi$ c.



April/May 2015

1. Design a one way, reinforced concrete slab simply supported at the edges for a public building with a clear span of 4m supported on 200mm solid concrete masonry walls. Live load on slab is  $5 \text{ kN/m}^2$ . Adopt M20 grade concrete and Fe415 HYSD bars.

Given data

clear span = 4m ; width of support = 200mm ; Live load =  $5 \text{ kN/m}^2$   
Floor finish =  $1.0 \text{ kN/m}^2$  ;  $f_{ck} = 20 \text{ N/mm}^2$  ;  $f_y = 415 \text{ N/mm}^2$

Depth of slab

Assume,  $d = \frac{\text{span}}{25} = 160 \text{ mm}$

Assume clear cover = 20mm  
Assume 10mm # bar ;

eff. depth =  $d = 140 \text{ mm}$   
 $D = 165 \text{ mm}$

(i) eff. span = clear span + eff depth  
 $= 3.5 + 0.14 = 3.64 \text{ m}$

(ii)  $\frac{1}{4}$  of support =  $3.5 + 0.2 = 3.7 \text{ m}$

Take  $L = 3.64 \text{ m}$ .

Load calculation

self wt of slab =  $b \cdot D \cdot \gamma_c = 4 \cdot 125 \text{ kN/m}$

F.F.L =  $1.00 \text{ kN/m}$

L.L =  $4.00 \text{ kN/m}$

Total service load =  $9.125 \text{ kN/m}$

$W_u = 13.69 \text{ kN/m}$

Ultimate moment  $M_u = \frac{W_u L^2}{8} = 22.67 \text{ kN}\cdot\text{m}$

Shear force  $V_u = \frac{W_u \cdot L}{2} = 24.92 \text{ kN}$

$M_{\text{limit}} = 54 \text{ kN}\cdot\text{m} > M_u$  ( $\therefore$  under reinforced)

$A_{st} = 480 \text{ mm}^2$

Provide 10mm # @ 160mm c/c.

$A_{st \text{ min}} = 0.12\% \cdot bD = 198 \text{ mm}^2$

Distribution, 8mm # @ 250mm c/c.

$T_v = \frac{V_u}{bd} = 0.178 \text{ N/mm}^2$  ;  $\frac{100 A_{st}}{bd} = 0.17$

$T_c = 0.35 \text{ N/mm}^2 > T_v$  ( $\therefore$  Shear within the Permissible limit)

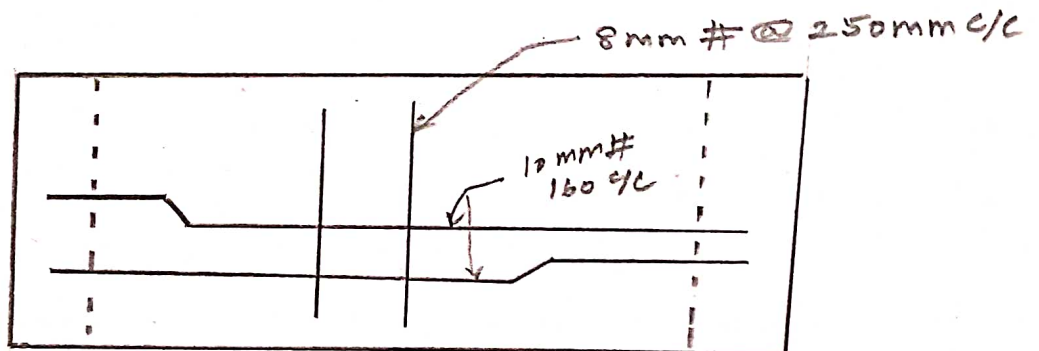
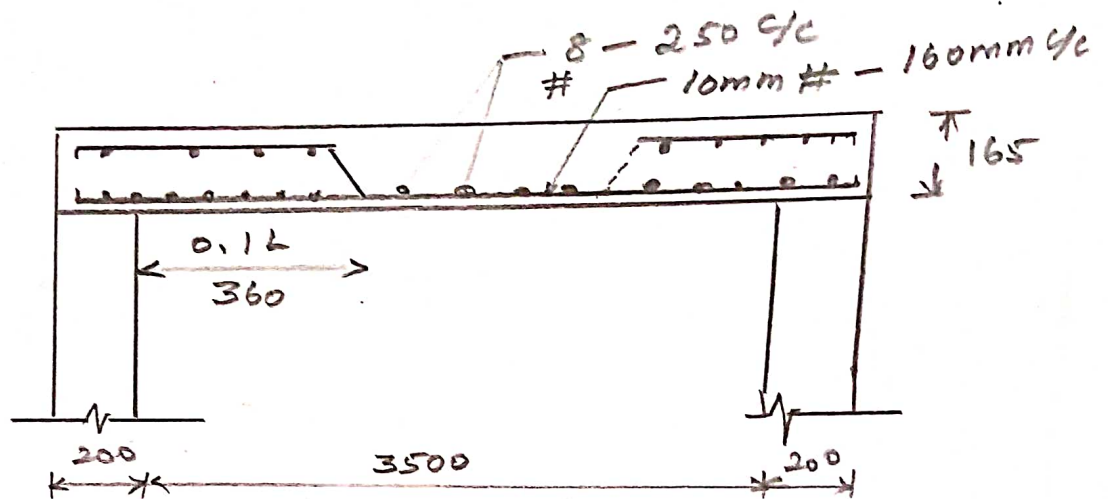
Check for deflection

$$\left(\frac{L}{d}\right)_{\max} = \left(\frac{L}{d}\right)_{\text{basic}} \cdot k_E \cdot k_c \cdot k_f = 28$$

$$\left(\frac{L}{d}\right)_{\text{act}} = 26$$

$$\therefore \left(\frac{L}{d}\right)_{\max} > \left(\frac{L}{d}\right)_{\text{act}}$$

Hence safe.

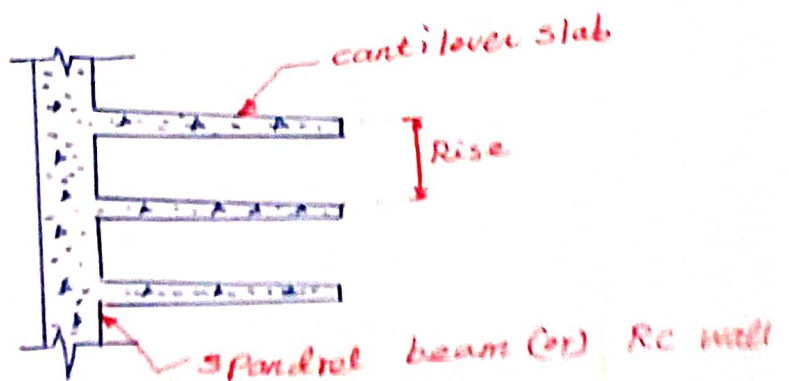
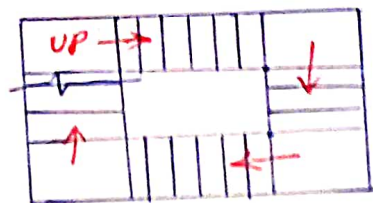
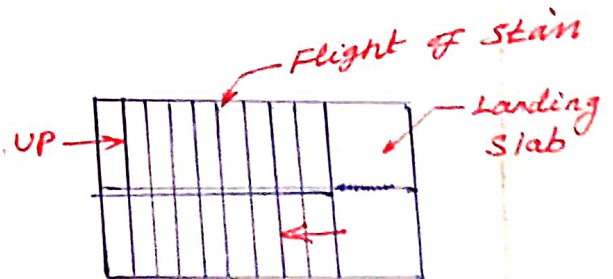


## Design of Staircase

- \* Staircases are provided connecting successive floors of a building.
- \* Staircase comprises of flight of steps generally one or more intermediate landings provided b/w the floor levels.
- \* Tread  $\rightarrow$  horizontal portion of step  
 $\rightarrow$  250 mm to 300 mm wide (based on type of building)
- \* Riser  $\rightarrow$  vertical distance b/w the adjacent treads  
 $\rightarrow$  150 mm to 190 mm
- \* Width of stair  $\rightarrow$  1 to 1.5 m  
 $\rightarrow$  minimum 0.85 m.
- \* A flight of steps consists of two landings & one 'going' with 10 to 12 steps.

### Types of Staircase:-

1. Dog-legged staircase
2. Open well staircase
3. Tread riser staircase
4. Cantilever staircase
5. Double cantilever precast tread slab staircase



## Load on staircase

- \* Dead loads  $\rightarrow$  include self wt of stair slab/waist slab, tread & risers & self wt of finishes
- \* Live loads  $\rightarrow$  IS: 875-1987 (Part-II)  
For Residential building  $\rightarrow$  UDL = 2 to 3  $\text{kN/m}^2$   
For public building  $\rightarrow$  5  $\text{kN/m}^2$

## Effective span of stair:-

IS: 456-2000; P.No: 63, Cl: 33.1

## Distribution of loading on stair

IS: 456-2000; P.No: 63; Cl: 33.2



1. Design one of the flights of a dog-legged stairs spanning b/w landing beams using the following data

Types of stair cases: dog-legged with waist slab, treads and risers

Number of steps in the flight = 10

Tread (T) = 300 mm

Rise (R) = 150 mm

width of landing beams = 300 mm

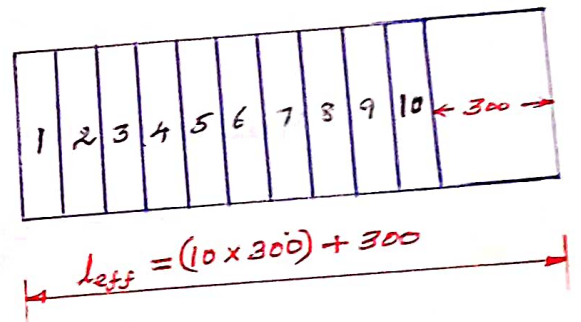
M 20 grade concrete ( $f_{ck} = 20 \text{ N/mm}^2$ )

Fe 415 HYSD bars ( $f_y = 415 \text{ N/mm}^2$ )

Step: 1 - Given Data:-

Step: 2 - Effective span:-

$$\begin{aligned} (i) \quad l_{\text{eff}} &= (10 \times 300) + 300 \\ &= 3000 + 300 \\ &= 3300 \text{ mm} \end{aligned}$$



(ii) Thickness of waist slab:-

$$\frac{l}{D} = 20$$

$$\therefore D = \frac{3300}{20} = 165 \text{ mm}$$

Overall depth (D) = 165 mm

$$\text{effective depth (d)} = D - d' - \frac{\phi}{2} = 165 - 15 - \frac{10}{2}$$

$$\boxed{d = 140 \text{ mm}}$$

Step: 3 - Load calculation:-

Dead load of slab on slope ( $W_s$ ) =  $0.165 \times 1 \times 25 = 4.125 \text{ kN/m}$

Dead load of slab on horizontal span is

$$W = \frac{W_s \sqrt{R^2 + T^2}}{T} = \frac{4.125 \times \sqrt{(150)^2 + (300)^2}}{300}$$

$$= 4.61 \text{ kN/m}$$

$$\text{Dead load of one step} = (0.5 \times 0.15 \times 0.3 \times 25) = 0.56 \text{ kN/m}$$

$$\text{Load of steps per metre length} = \frac{0.56 \times 1000}{300} = 1.86 \text{ kN/m}$$

$$\text{Finishes (Assume)} = 0.53 \text{ kN/m}$$

$$\therefore \text{Total } \overset{\text{Dead}}{\text{load}} = 4.61 + 1.86 + 0.53 = 7 \text{ kN/m}$$

$$\text{Service Live load} = 5 \text{ kN/m}^2 \text{ (Assume)}$$

$$\therefore \text{Total load} = \text{DL} + \text{L.L} = 7 + 5 = 12 \text{ kN/m}$$

$$\therefore \text{Factored load (Wu)} = 1.5 \times 12 = 18 \text{ kN/m}$$

#### Step: 4 - Bending Moment

Maximum bending moment at centre of span

$$M = \frac{W_u l^2}{8} = \frac{18 \times 3.3^2}{8} = 24.5 \text{ kN.m.}$$

#### Step: 5 - Check for depth of waist slab:

IS: 456: 2000 - P. NO: 96, G. 1.1 (C)

$$M_{u \text{ lim}} = 0.36 \frac{x_{u \text{ max}}}{d} \left(1 - 0.42 \frac{x_{u \text{ max}}}{d}\right) f_{ck} b d^2$$

$$24.5 \times 10^6 = 0.36 \times 0.48 \left(1 - [0.42 \times 0.48]\right) \times 20 \times 1000 \times d^2$$

$$\therefore d = 94.22 \text{ mm} < 140 \text{ mm}$$

Hence safe.

#### Step: 6 - Main Reinforcement

IS: 456 - 2000 ; P. NO: 96, G. 1.1 (b)

$$M_u = 0.87 f_y A_{st} d \left[1 - \left(\frac{f_y A_{st}}{f_{ck} b d}\right)\right]$$

$$24.5 \times 10^6 = 0.87 \times 415 A_{st} \times 140 \left[1 - \left(\frac{415 A_{st}}{20 \times 1000 \times 140}\right)\right]$$

Solving,

$$A_{st} = 530 \text{ mm}^2 \text{ per metre.}$$

Assume 12mm  $\phi$

Bar/area =  $\frac{1000}{s}$

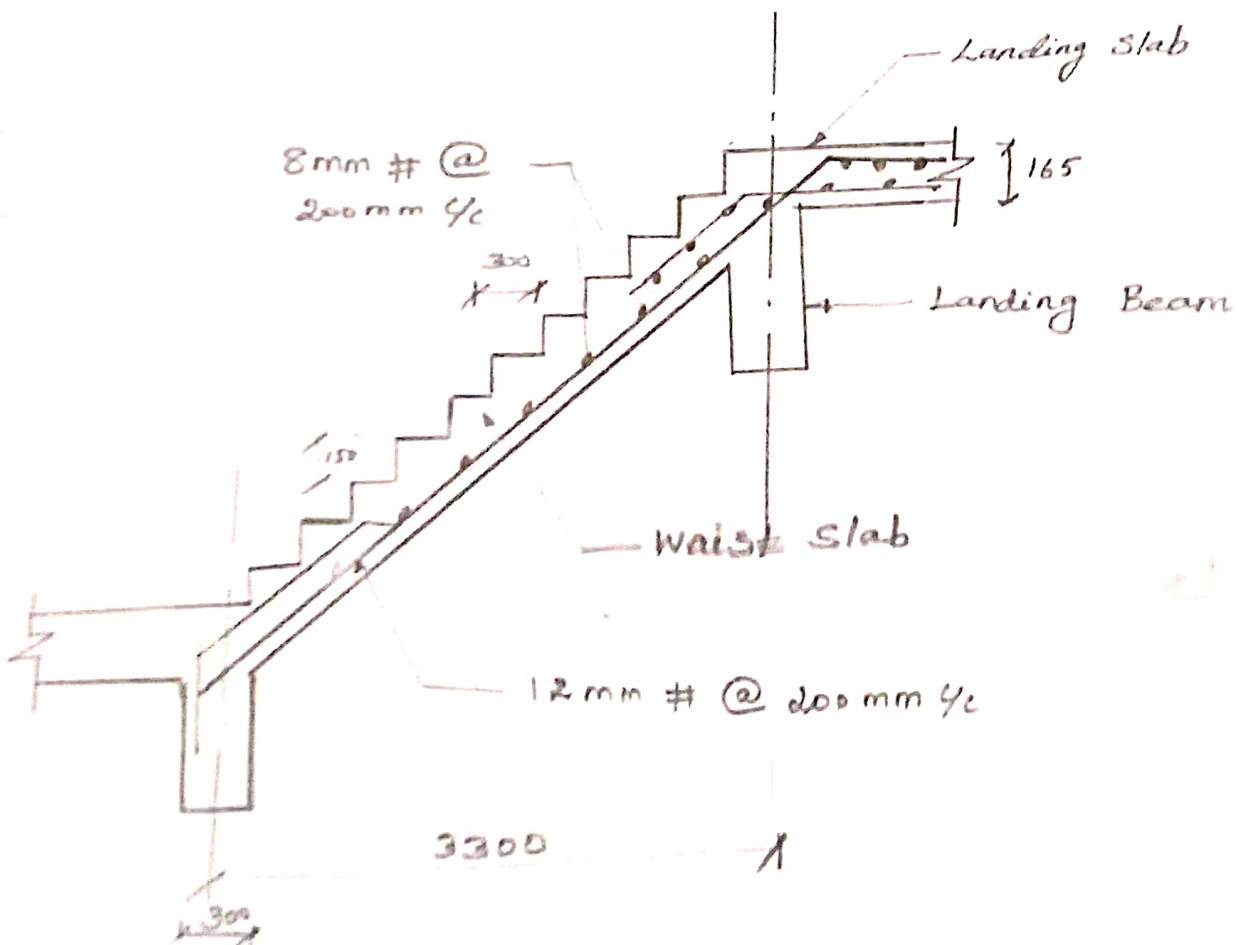
Provide 12mm dia. bars @ 200mm  $\phi$  ( $A_{st} = 565 \text{ mm}^2$ )

Step: 7 - Distribution Reinforcement

$$A_{st \text{ min}} = 0.12 \% b D = \frac{0.12}{100} \times 1000 \times 165 = 198 \text{ mm}^2 \text{ per metre}$$

Provide 8mm dia @ 200mm  $\phi$  ( $A_{st} = 251 \text{ mm}^2$ ).

Step: 8 - Reinforcement Details :-



## Unit - IV

### Limit state design of column

Column  $\rightarrow$  a structural member subjected to compression force in a direction parallel to its longitudinal axis.  
(or) Stanchions  
(or) post

Strut  $\rightarrow$  a structural member subjected to compressive force in a direction parallel to its longitudinal axis.

\* used for compression member in roof trusses.

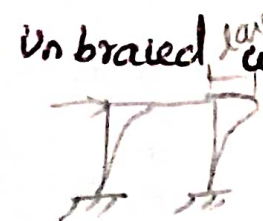
Slenderness Ratio ( $\lambda$ )  $\rightarrow$  ratio of eff. length to least lateral dimensions

$$\lambda = \frac{l_{eff}}{\gamma_{min}} \quad (or) \quad \frac{l_{eff}}{b}$$

Braced column  $\rightarrow$  In certain situations, column may be subjected to lateral loads like wind or earthquake forces. Thus the column have to be planned to sustain the lateral loads. This is done by joining the column by structure called bracings (or) by providing a shear wall.



Unbraced column  $\rightarrow$  ones which are to resist the lateral load in addition to vertical load.



Types of column

- Based on length / slenderness ratio
- Type of loading
- Type of reinforcement
- Shape of column (rectangular, square, circular)

Based on slenderness ratio

Short column  $\rightarrow$   $\left( \frac{l_{eff}}{\gamma} \text{ less than } 12 \right)$  — P. NO: 41 ; 25.1.2

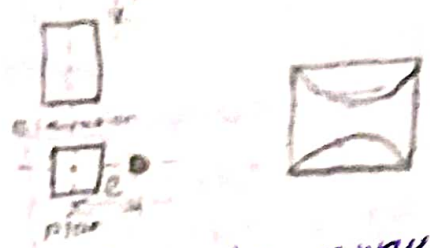
Long/slender column  $\rightarrow$   $\left( \frac{l_{eff}}{\gamma} \text{ greater than } 12 \right)$  IS: 456: 2000

PNB 71 ; CI 39-3 12.456  
Based on Type of Loading



**Axial loading** → loading made along the centroidal axis.  
 (Axially loaded column)  
 \* In practice pure axial comp. does not occur.  
 \* Bending sometimes occurs.

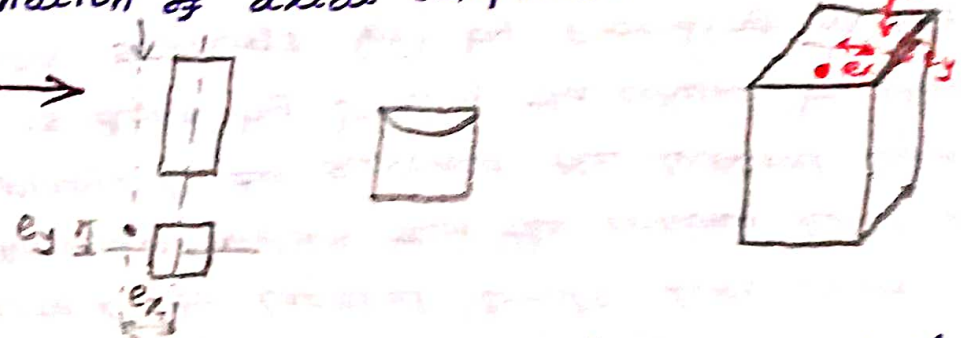
**Uniaxial loading** →



\* If the load acts away from the centroidal axis but along X-axis (or) Y-axis it is called Uniaxial loading

\* combination of axial compression with bending  $M_x$  &  $M_y$

**Bi-axial loading** →

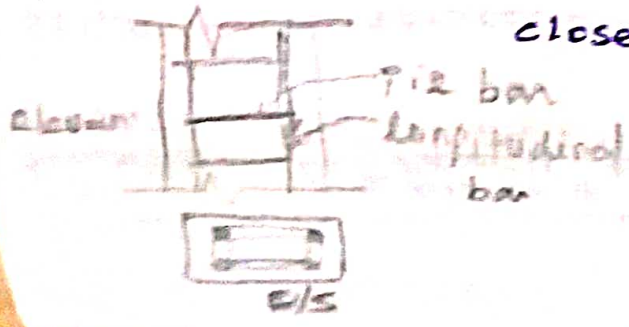


\* If the load acts away from the centroidal axis & also not acting along X-axis or Y-axis. it is called bi-axial bending.

\* combination of axial compression with bending on both directions  $M_x$  &  $M_y$ .

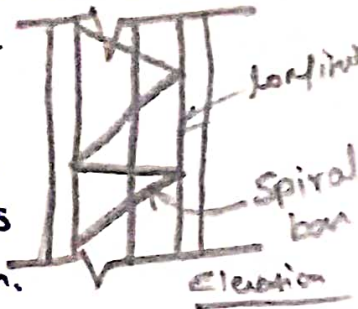
Based on Type of Reinforcement

**Tied column** → main longitudinal bars are enclosed with in closely spaced lateral ties.



**Spiral column** → The main longitudinal bars are enclosed within spiral reinforcement.

**Composite column** → The reinforcement is in the form of structural sections or pipes with or without longitudinal bar.



## Assumptions in columns

IS: 456-2000; P-NO: 70

Cl: 39.1

- \* max. compr. strain in concrete in axial comp is taken as 0.002
- \* max. comp. strain at the highly compressed extreme fibre in concrete subjected to axial compr. & bending & when there is no tension on the section shall be (0.0035 - 0.75 times the strain at the least compressed extreme fibre)

IS: 456: 2000 - P-NO: 69 ; Cl: 38.1

## Modes of failure of axially loaded column.

**Pure compression failure** → Failure of the column takes place under axial load without any lateral deformation. Steel & concrete reach the yield stress values at the same time. The collapse of the column is due to material failure.

**Combined compression & Bending failure.**

Short columns are subjected to a direct load (P) & Moment (M)

Slender/long column undergo deflection along length even loaded axially.

**Failure by elastic instability** Such deflection produce additional moment in column.

- \* Very long column may lose stability even under small loads well before the materials reach yield stresses. member fails by lateral elastic buckling.

Further the failure of the material is reached under the combined action of these direct loads & BM.

- \* Under practice — is not applicable.

## Loads on columns

Live load — floor ~~beam~~ supported by column

Dead load — floor & beam

Self wt of column

Eff. length of compression member End condition,

IS: 456:2000; P.NO: 94; Table: 28

Minimum Eccentricity

IS: 456:2000; P.NO: 76; Cl: 39.2

IS: 456:2000; P.NO: 42; Cl: 25.4

Design of short axially loaded columns.

- \* min. reinforcement  $\rightarrow$  12 mm  $\phi$  bar — P.NO: 46  
Cl: 26.4.2.1
- x % of longitudinal bar  $\rightarrow$  min 0.8%  $A_g$ ; max  $\rightarrow$  6%  $A_g$   
P.NO: 48; Cl: 26.5.3.1
- \* Normally should not exceed 4%
- \* min. cover 40 mm. — P.NO: 46 Cl: 26.4.2.1
- \* spacing — not exceed 300 mm P.NO: 48  
26.5.3.1
- \* Rectangular, square column  $\rightarrow$  min. 4 rod
- \* circular column  $\rightarrow$  min 6 nos.

① Design an axially loaded tied column  $400 \times 400$  mm pinned at both ends with an unsupported length of 3m to carry a factored load of 2300 kN. Use M20 grade concrete & Fe415 steel. (NOV/DEC 2007)

Given data

Unbraced length  $L_{eff} = 3$  m ; size of column =  $400 \times 400$  mm  
 Factored load = 2300 kN  
 $f_{ck} = 20$  N/mm<sup>2</sup>;  $f_y = 415$  N/mm<sup>2</sup>

Condition: Pinned at both end  $L = L_{eff} = 3$  m.

Step 2 - Slenderness ratio

$$\lambda = \frac{L_{eff}}{D} = \frac{3000}{400} = 7.5 < 12 \quad (\text{Hence the column is designed as a short column})$$

Step 3 - Minimum eccentricity

IS: 456-2000; P. NO: 42; CI: 25.4  
 lateral dimensions

$$e_{min} = \frac{L}{500} + \frac{D}{30} = \frac{3000}{500} + \frac{400}{30} = 19.33 < 20$$

Hence the eccentricity condition is satisfied.

$0.05D = 0.05 \times 400 = 20$

Step 4 - Main reinforcement

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

$$2300 \times 10^3 = 0.4 \times 20 A_c + 0.67 \times 415 A_{sc}$$

$$2300 \times 10^3 = 8 A_c + 278.05 A_{sc}$$

$$2300 \times 10^3 = 8 (A_g - A_{sc}) + 278.05 A_{sc}$$

$$= 8 (400 \times 400 - A_{sc}) + 278.05 A_{sc}$$

$$= 1280 \times 10^3 - 8 A_{sc} + 278.05 A_{sc}$$

$$2300 \times 10^3 = 1280 \times 10^3 = 278.05 A_{sc}$$

$$A_{sc} = 3777 \text{ mm}^2$$

$$A_c = A_g - A_{sc}$$

$$A_g = bD \quad (\square_{len})$$

$$A_g = \pi r^2 \quad (O_{dia})$$

Percentage of steel  $p = \frac{100 A_{sc}}{bD} = \frac{100 \times 3777}{400 \times 400} = 2.36\%$

(min 0.8% + max 6%) Hence safe.



No. of rods  $n = \frac{A_{st}}{A_{\phi}} = \frac{3777}{\pi \times 25^2 / 4} = 7.69 \text{ nos} \approx 8$

Assume 25mm  $\phi$

Hence provide 8 nos of 25mm  $\phi$

$A_{sc} = 8 \times \pi \times 25^2 / 4$   
 $A_{sc} = 3927 \text{ mm}^2$

Lateral ties IS: 456-2000; P.No: 49

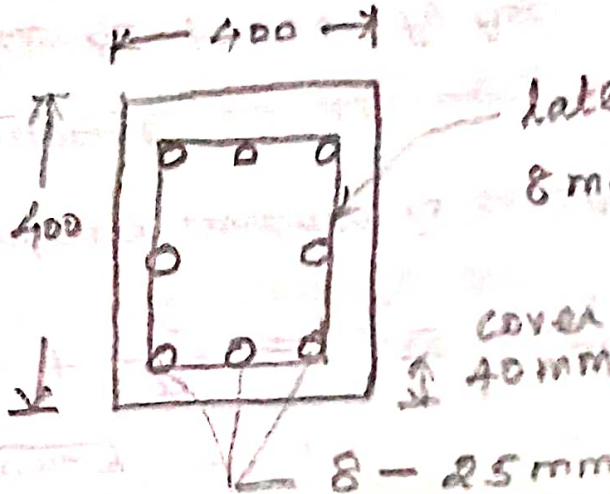
- (i)  $\frac{1}{4} \phi$  of main bar =  $\frac{1}{4} \times 25 = 6.25 \approx 8 \text{ mm}$  } Take greater value  
 (ii) 5 mm

Spacing of ties

- (i) dimensions of column  $b = 400 \text{ mm}$   
 (ii) 16  $\phi$  main bar =  $16 \times 25 = 400 \text{ mm}$   
 (iii) 48  $\phi$  lateral ties =  $48 \times 8 = 384 \text{ mm}$   
 (iv) 300 mm

Take least value  
 $V_1 = 400$  (circle)  
 $V_2 = 384$  (square)  
 $V = \sqrt{V_1^2 - V_2^2}$

lateral links/ties  
 8 mm # @ 300 mm / c



8 - 25 mm # main reinforcement

Q2 design the required reinforcement in a column of 400x600mm size subjected to a characteristic axial load of 2000 kN. The column has an unsupported length of 3m and is braced against the sidesway in both dimensions. Use M20 + Fe15 grades. (Nov/Dec 2009)

Given Data

Unsupported length  $L_0 = 3 \text{ m}$

size of column = 400 x 600 mm  
 characteristic axial load = 2000 kN

$\therefore$  Factored load  $P_u = 1.5 \times 2000 = 3000 \text{ kN}$

Condition: Braced against sidesway,  $l_{eff} = L = 3 \text{ m}$

Slenderness ratio ( $\lambda$ )  $\lambda = \frac{l_{eff}}{D} = \frac{3000}{400} = 7.5 < 12$  ( $\therefore$  The column is designed as a short column)

Min. eccentricity: IS: 456; P.No: 42; Cl: 25.4 (or) SP: 16 - P.No: 99  
 $e_{min} = \frac{l_{eff}}{500} + \frac{D}{20} = \frac{3000}{500} + \frac{400}{20} = 19.33 < 20$   
 $0.05D = 0.05 \times 400 = 20$

Hence eccentricity condition is satisfied.

Main reinforcement SP: 16; P. NO: 99 (or) IS: 456-2000; P. No: 71; Cl: 39.3

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

$$A_c = A_g - A_{sc} = 400 \times 600 - A_{sc}$$

$$3000 \times 10 = 0.4 \times 20 \times (400 \times 600 - A_{sc}) + 0.67 \times 415 A_{sc}$$

$$A_{sc} = 3999 \text{ mm}^2$$

% of steel

$$p = \frac{100 A_{sc}}{b D} = \frac{100 \times 3999}{400 \times 600} = 1.67\% \quad \left( \begin{array}{l} \text{min } 0.8\% \\ \text{max } 6\% \end{array} \right)$$

Hence OK.

No. of bars

Assume 25 mm #

$$n = \frac{A_{sc}}{A_p} = \frac{3999}{\pi \times 25^2 / 4} = 8.15 \text{ NOS}$$

8 - 25 mm #  
2 - 16 mm #

Provide 8 - NOS of 25 mm # & 2 nos of 16 mm #

$$A_{sc} = \left[ \pi \times \frac{25^2}{4} \times 8 \right] + \left[ \pi \times \frac{16^2}{4} \times 2 \right] = 4329 \text{ mm}^2$$

Lateral ties/links

IS: 456-2000 - P. NO: 49

(i)  $\frac{\text{Length dia of main bar}}{4} = \frac{25}{4} = 6.25 \approx 8 \text{ mm}$

(ii) 5 mm

Hence provide 10 mm #

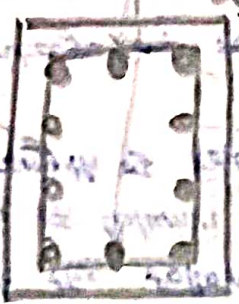
25 mm # not sufficient  
5 mm sufficient  
greater value

spacing

- (i) least lateral dimensions = 400 mm
  - (ii) 16 times dia smallest main bar =  $16 \times 16 = 256 \text{ mm} \approx 250 \text{ mm}$
  - (iii) 300 mm
- Take least value, spacing = 250 mm

Hence provide lateral ties at 10 mm # @ 250 mm c/c

lateral ties  
10 mm # @  
250 mm c/c



2 - 16 mm #

8 - 25 mm #

**HW** determine the cross section & the reinforcement for an axially loaded column with the following data. Factored load = 3000 kN  
 unsupported length of column = 3m; concrete grade = M20 & characteristic strength of reinforcement = 415 N/mm<sup>2</sup> [Anand Univ. Apr/May 2011]

solution

Assume Percentage of reinforcement  $p = 2\%$

$$\therefore A_{sc} = 2\% A_g = \frac{2}{100} A_g = 0.02 A_g$$

$$A_c = A_g - A_{sc} = A_g - 0.02 A_g = 0.98 A_g$$

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

$$3000 \times 10^3 = 0.4 \times 20 \times 0.98 A_g + 0.67 \times 415 \times 0.02 A_g$$

$$A_g = 223864 \text{ mm}^2$$

Assume  $b = D/2$

Assume square column

$$A_g = b \cdot D$$

$$223864 = \frac{D}{2} \cdot D$$

$$\therefore D = \sqrt{2 \times 223864} = 669.14 \text{ mm} \approx 500 \text{ mm}$$

$$D = 500 \text{ mm}$$

$$b = 400 \text{ mm}$$

size of column is  $400 \times 500 \text{ mm}$

check for eccentricity

$$\frac{e_{\text{eff}}}{500} + \frac{b}{30} = \frac{3000}{500} + \frac{400}{30} = 19.33 < \frac{0.05 \times 1500}{0.05 \times 400}$$

Hence OK.

Check for short/long column

$$\text{slenderness ratio } \lambda = \frac{e_{\text{eff}}}{b} = \frac{3000}{400} = 7.5 < 12$$

Hence it is a short column.

$A_{sc}$

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc} = 0.4 \times 20 \times (A_g - A_{sc}) + 0.67 \times 415 A_{sc}$$

$$3000 \times 10^3 = 8 \times (600 \times 400 - A_{sc}) + 0.67 \times 415 A_{sc}$$

$$A_{sc} = 3999 \text{ mm}^2$$

H.W. design a <sup>CIRCULAR</sup> rectangular column to carry an axial load of 1200 kN. The column has an unsupported length of 4.5 m and is restrained and direction at both ends. Use M20 + Fe415. Same as previous

$$A_g = \pi d^2 / 4$$

Changes

$$P_u = 1.5 \times 1200 = 1800 \text{ kN} ; L = 4.5 \text{ m}$$

End condition: restrained and direction at both end (Both end fixed)

$$\therefore L_{eff} = 0.65 L = 0.65 \times 4.5 = 2.925 \text{ m.}$$

Substitute  $L_{eff}$  → eccentricity, slenderness ratio

Circular column with helical reinforcement

Design a circular column of diameter 400 mm with helical reinforcement subjected to a working load of 1200 kN. The column has unsupported length of 3 m and is effectively held in position at both end but not restrained against rotation. Assume M25 grade of concrete & Fe415 steel. Adopt limit state method [NOV/DEC 2010]

Given Data

Dia of column ( $\phi$ ) = 400 mm  
Working load  $P = 1200 \text{ kN}$

$$P_u = 1.5 \times 1200$$

factored load  $P_u = 1800 \text{ kN}$

Unsupported length ( $L$ ) = 3 m

$$f_{ck} = 25 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

End condition: Effectively held in position at both end but not restrained against rotation

IS: 456-2000 P.NO: 94  
Table: 28

$$L_{eff} = L = 3 \text{ m.}$$

Slenderness ratio ( $\lambda$ )

$$\lambda = \frac{L_{eff}}{D} = \frac{3000}{400} = 7.5 < 12$$

Hence, it is a short column.

minimum eccentricity

SP: 16  
P.NO: 99 (or)

IS: 456-2000; P.NO: 42

$$e_{min} = \frac{L_{eff}}{500} + \frac{\phi}{30}$$

$$= \frac{3000}{500} + \frac{400}{30}$$

$$e_{min} = 19.33 \text{ mm} < 20 \text{ mm}$$

$$\neq 0.05 D$$

$$= 0.05 \times 400$$

$$= 20 \text{ mm}$$

Hence min. eccentricity is acceptable,

Main reinforcement

IS: 456-2000; P.NO: 71

$$c1: 39.4$$

$$P_{u(helical)} = 1.05 P_u$$

$$P_{u(helical)} = 1.05 [0.4 f_{ck} A_c + 0.67 f_y A_{sc}] \quad \text{--- (1)}$$

$$A_c = A_g - A_{sc}$$

$$A_c = \frac{\pi \times 400^2}{4} - A_{sc}$$

$$\therefore (1) \Rightarrow \frac{1800 \times 10^3}{1.05} = 0.4 \times 25 \times \left( \frac{\pi \times 400^2}{4} - A_{sc} \right) + 0.67 \times 415 A_{sc}$$

$$1714.3 \times 10^3 = 125663 + 268.05 A_{sc}$$

$$A_{sc} = 1708 \text{ mm}^2$$

$$p = \frac{100 A_{sc}}{bD} = \frac{100 \times 1708}{400 \times 400}$$

$$= 1.07\%$$

$$[\text{min } A_{sc} = 0.8\%]$$

$$\text{max } A_{sc} = 6\%$$

$$P_{provided} = 1.07\% \text{ hence OK.}$$

No. of bars

Assume 20mm #

$$n = \frac{A_{st}}{A_{\phi}} = \frac{1708}{\pi \times 20^2 / 4} = 5.43 \text{ nos} \approx 6 \text{ nos.}$$

$$A_{sc} = 6 \times \pi \times 20^2 / 4$$

$$A_{sc} = 1884 \text{ mm}^2$$

Provide 6 nos of 20mm #

### Helical reinforcement

Adopt clear cover = 50mm over spiral

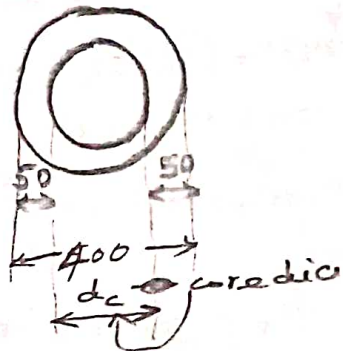
(2) core diameter ( $d_c$ ) =  $400 - (2 \times 50) = 300 \text{ mm}$

$$\text{Area of core} = \frac{\pi \times 300^2}{4} = 1884 \text{ mm}^2$$

$$A_c = 68801 \text{ mm}^2$$

$$\text{Volume of core/m} (V_c) = 68801 \times 10^3 \text{ mm}^3$$

$$\text{Gross area of section} (A_g) = \frac{\pi \times 400^2}{4} = 125664 \text{ mm}^2$$



### Transverse (Helical) reinforcement

- (a)  $\phi$  of spiral/helical IS: 456-2000 P.NO: 49
- (i)  $\frac{1}{4}$  main bar =  $\frac{1}{4} \times 20 = 5 \text{ mm} \approx 6 \text{ mm}$
  - (ii) not more than 16mm

Take 8mm #

- (b) spacing IS: 456-2000 P.NO: 49
- (i) not more than 75mm
  - (ii)  $\frac{1}{6} d_c = \frac{1}{6} \times 300 = 50 \text{ mm}$  not less than
  - (iii) not less than 25mm
  - (iv)  $3 \times \phi$  of spiral =  $3 \times 8 = 24 \text{ mm}$

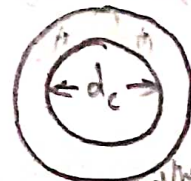
Hence provide 8mm # @ 50mm  $\phi$

$$b^2 = 1.2 \times 1800 = 1800 \text{ mm}^2 ; T = 1.2 \text{ m}$$

IS: 456-2000 ; P.NO: 71 ; clause: 39.4.1

$$\frac{V_h}{V_c} \leq 0.36 \left( \frac{A_g - A_c}{A_c} \right) \frac{f_{ck}}{f_y}$$

$$V_h = 1146681 \text{ mm}^3 = \pi \times [(300 - 8) \times 50 \times \frac{1000}{40}] \text{ pitch}$$

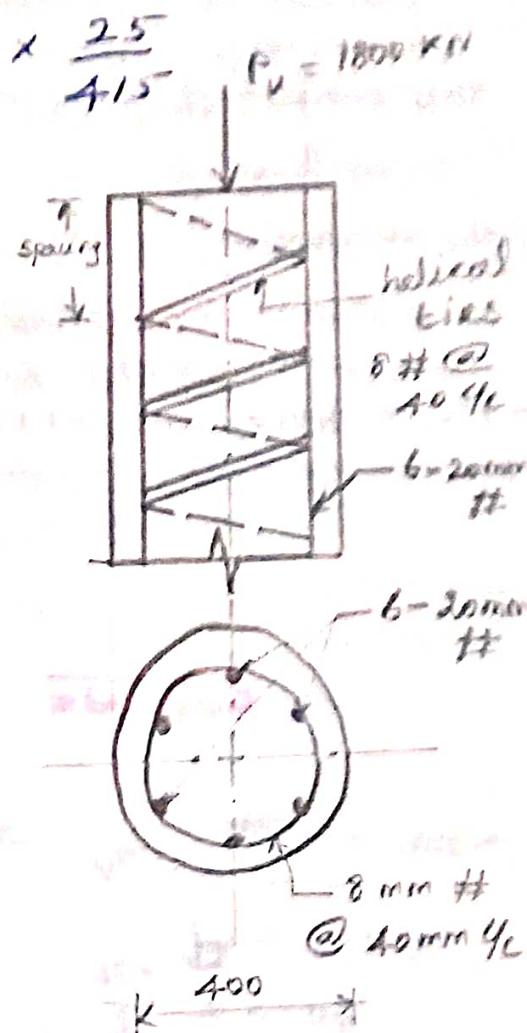


$$\frac{1146681}{68801 \times 10^3} \neq 0.36 \times \left( \frac{70686}{68801} - 1 \right) \times \frac{25}{415}$$

$$0.017 \neq 5.94 \times 10^{-4}$$

Hence safe.

Hence provide 8 mm # @ 40 mm c/c.



### Uniaxial loading column

1) design the reinforcements in a rectangular column of size 300mm x 500mm to support a design ultimate load of 500kN, together with a factored moment of 200 kN.m. Adopt M20 grade of concrete & Fe 415 HYSP bars.

#### Given data

$$\begin{aligned} P_u &= 500 \text{ kN} \\ b &= 300 \text{ mm} \\ D &= 500 \text{ mm} \\ M_u &= 200 \text{ kN}\cdot\text{m} \\ f_{ck} &= 20 \text{ N/mm}^2 \\ f_y &= 415 \text{ N/mm}^2 \end{aligned}$$

Step: 2 - Non Dimensional Parameters

Assume effective cover ( $d'$ ) = 40 mm

$$(i) \frac{d'}{D} = \frac{40}{500} = 0.08 \approx 0.1$$

$$(ii) \frac{P_u}{f_{ck} \cdot b \cdot d} = \frac{500 \times 10^3}{20 \times 300 \times 500} = 0.166$$

$$(iii) \frac{M_u}{f_{ck} \cdot b \cdot D^2} = \frac{200 \times 10^6}{20 \times 300 \times 500^2} = 0.132$$

Step: 3 - Longitudinal reinforcement

SP: 16-1978 ; P.No: 117; Chart- 32

based on  $\left( \frac{d'}{D} + f_y \text{ value} \right)$  ←  
Chart selection

$$\frac{P}{f_{ck}} = 0.06$$

$$p = 0.06 \times 20$$

$$p = 1.2\%$$

$$A_{sc} = \frac{100 P_{sc}}{bD}$$

$$A_{sc} = \frac{p \cdot b \cdot D}{100}$$

$$A_{sc} = 1800 \text{ mm}^2$$

No. of bars (n)

$$n = \frac{A_{sc}}{A_{\phi}}$$

Assume 25mm  $\phi$

$$n = 4 \text{ nos}$$

$$A_{sc} = 1964 \text{ mm}^2$$

### Step: 1 - Lateral ties

(a) Diameter

(i)  $\frac{1}{4}$  main dia = 6.25

(ii) 6mm

Take greater value

$\therefore$  Provide 8mm  $\phi$

(b) Spacing

(i) 16  $\phi$  main bar

(ii) 48  $\phi$  ties

(iii) 300

Take least value = 300mm

Provide 8mm # @ 300mm  $\phi$  lateral ties

2) Design the reinforcements in a circular column of dia 400mm to support a factored load of 800 kN together with a factored moment of 80 kN.m. Adopt M20 grade of concrete & Fe 415 HYSD Bars.

### Step: 1 - Given data

$$D = 400 \text{ mm}$$

$$P_u = 800 \text{ kN}$$

$$M_u = 80 \text{ kN.m}$$

Assume  $d' = 40 \text{ mm}$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

### Step: 2 - Non dimensional Parameters

$$(i) \frac{d'}{D} = \frac{40}{400} = 0.1$$

$$(ii) \frac{P_u}{f_{ck} \cdot D^2} = \frac{800 \times 10^3}{20 \times (400)^2} = 0.25$$

$$(iii) \frac{M_u}{f_{ck} \cdot D^3} = \frac{80 \times 10^6}{20 \times (400)^3} = 0.0625$$

### Step: 3 - Longitudinal reinforcement

SP: 16-1978; P.NO: 41

Chart: 56

$$\frac{p}{f_{ck}} = 0.06$$

$$P = 1.2\%$$

$$A_{sc} = \frac{p \cdot \pi D^2}{400} = 1508 \text{ mm}^2$$

$$\text{No. of bars (n)} = \frac{A_{sc}}{A_{\phi}}$$

Assume 20mm #

$$n = 6 \text{ nos}$$

### Lateral ties

#### Tie dia

Same as previous Problem.

### spacing

Same as previous Problem

3) A column size 300mm x 400mm has effective length of 3.6m and is subjected to a factored load of 1100kN and a factored moment of 150 kNm about the major axis. Design the column using M25 & Fe 415.

1) Given data

2) Check for slenderness ratio

$$\lambda = \frac{l_{eff}}{D} < 12 \text{ Short}$$

3) Check for eccentricity

$$e_{min} = \left( \frac{l}{500} + \frac{D}{30} \right) < 20 \text{ mm}$$

0.05D

4) Dimensional parameters

5) Longitudinal rebar

6) no. of bar

7) Lateral ties

a) diameter

b) spacing

April / May 2017

1. Design the reinforcements in a circular column of diameter 300mm to support a service axial load of 800 kN. The column has an unsupported length of 3m and is braced against side sway. The column is reinforced with helical ties. Adopt M20 grade concrete and Fe 415 HYSD bars.

Given data

$$D = 300 \text{ mm} ; P = 800 \text{ kN} ; P_u = 1200 \text{ kN} ; L = 3 \text{ m} ;$$

$$f_{ck} = 20 \text{ N/mm}^2 ; f_y = 415 \text{ N/mm}^2$$

slenderness ratio ( $\lambda$ ) =  $\frac{L_{eff}}{D} = 7.5 < 12$  ( $\therefore$  short column)

R.No: 42  $e_{min} = \frac{L_{eff}}{500} + \frac{D}{30} = 19.33 < (0.05D)^{20 \text{ mm}}$  Hence OK.

Main reinforcement

P.No: 71  
Cl: 394

$$P_{u, \text{helical}} = 1.05 P_u = 1.05 [0.4 f_{ck} A_c + 0.67 f_y A_{sc}]$$

$$A_c = A_g - A_{sc}$$

$$A_g = \frac{\pi D^2}{4} = 70.685 \times 10^3 \text{ mm}^2$$

$$A_{sc} = 2138.5 \text{ mm}^2$$

$$p = \frac{100 A_{sc}}{b D} = 2.38\%$$

$$p = 0.8 < 2.38 < 6\% \text{ Hence OK.}$$

Assume 25mm #

$$\text{No. of bars } (n) = \frac{A_{sc}}{A_{\phi}} = 5 \text{ nos}$$

Provide 5 nos of 25mm #

Helical Reinforcement

$$d_c = 300 - 2 \times 50 = 200 \text{ mm}$$

$$\text{Area of core} = A_g - A_{sc} = 28.961 \times 10^3 \text{ mm}^2$$

$$\text{Volume of core / m } V_c = 28.961 \times 10^3 \text{ mm}^3$$

$$A_g = 31.416 \times 10^3 \text{ mm}^2$$

Transverse (Helical reinforcement)

a)  $\phi$  of spiral (helical)

- (i)  $\frac{1}{4}$  main bar = 6.25mm } Take 8mm #  
 (ii) less than 16mm

(b) spacing

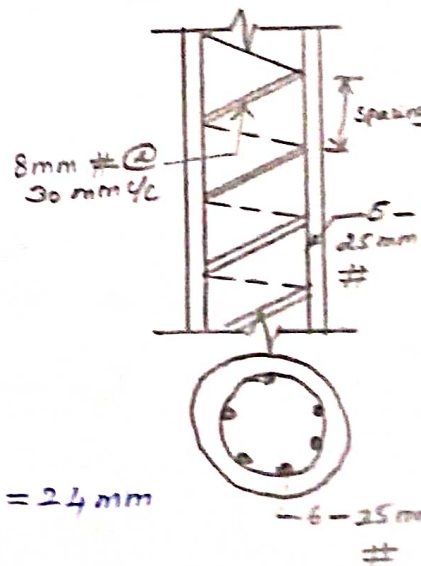
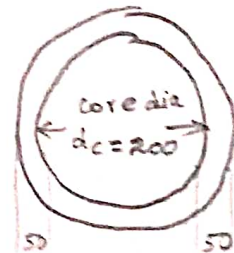
(i)  $\neq 75 \text{ mm}$

(ii)  $\frac{1}{6} d_c = 33.33 \text{ mm}$

(iii)  $\neq 25 \text{ mm}$

(iv)  $\neq 3 \times \phi$  of spiral = 24mm

Hence provide 8mm # @ 30mm  $\phi$ .





2. design the reinforcement in a short column 400x400mm at corner of a multistorey building to support an axial factored load of 1500kN together with biaxial moments of 50kNm acting in perpendicular planes. Adopt M20 grade concrete and Fe415 steel.

Given data

$b = 400\text{mm}$  ;  $D = 400\text{mm}$  ;  $f_{ck} = 20\text{N/mm}^2$  ;  $f_y = 415\text{N/mm}^2$

$P_u = 1500\text{kN}$  ; Assume,  $d' = 40\text{mm}$  ;  $M_{ux} = M_{uy} = 50\text{kN.m}$

$\frac{d'}{D} = 0.1$

Equivalent moment  $M_u = 1.15 \sqrt{M_{ux}^2 + M_{uy}^2} = 81.3\text{kN.m}$

Non-dimensional parameters

$\frac{P_u}{f_{ck} b D} = 0.468$

$\frac{M_u}{f_{ck} b D^2} = 0.063$

SP:16 - chart No: 44

corresponding to the values  $f_y = 415\text{N/mm}^2$  ;  $\frac{d'}{D} = 0.1$

$\frac{p}{f_{ck}} = 0.06$

$p = 1.2$   
 $A_{sc} = \frac{p b D}{100} = 1920\text{mm}^2$

Provide 4 - 20mm # & 4 - 16mm # ( $A_{sc} = 2060\text{mm}^2$ ) distributor and 4 - 16mm # ( $A_{sc} = 800\text{mm}^2$ ) distributed equally on all faces.

$p = \frac{100 A_{sc}}{b D} = 1.28$

$p/f_{ck} = 0.064$

From SP:16 - chart No: 44

corresponding to the values

$\frac{P_u}{f_{ck} b D} = 0.468$  &  $p/f_{ck} = 0.064$

$\frac{M_{ux}}{f_{ck} b D^2} = 0.468$

$M_{ux} = 0.468 f_{ck} b D^2 = 87\text{kN.m}$

Due to symmetry  $M_{ux} = M_{uy} = 87\text{kN.m}$

$P_{uz} = 0.45 f_{ck} A_c + 0.75 f_y A_{sc}$

$P_{uz} = 2062\text{kN}$

$\frac{P_u}{P_{uz}} = 0.72$

$\alpha_n = 1.8$

$A_c = A_g - A_{sc}$

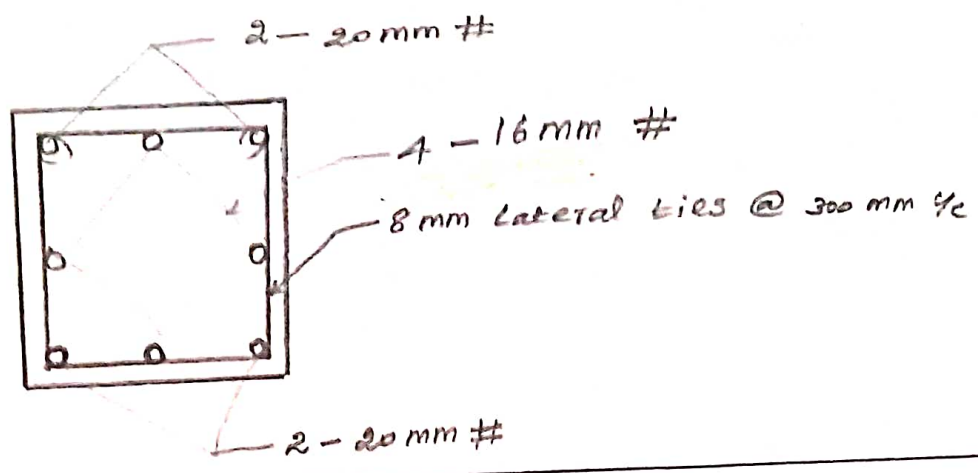
2060

check for safety under bi-axial bending

$$\left[ \left( \frac{M_{ux}}{M_{uxl}} \right)^{\alpha_n} + \left( \frac{M_{uy}}{M_{uyl}} \right)^{\alpha_n} \right] \leq 1$$

0.736 ≤ 1 (Hence the section is safe under bi-axial bending)

Reinforcement details



Nov/dec - 2016

1. Design a column having an effective length of 4.50m to support a factored load of 1600kN. Consider the reinforcement ratio  $p$  to be in the range 1.5 to 2 percent and the effective cover to longitudinal steel of 55mm. The materials to be used are M25 grade of concrete and HYSD steel bars of grade FE415.

Given data

$l_{eff} = 4.5m$  ;  $P_u = 1600kN$  ;  $p = 1.5 \text{ to } 2\%$  ;  $f_y = 415N/mm^2$   
 Take  $p = 2\%$  ; eff. cover = 55mm ;  $f_{ck} = 25N/mm^2$

$A_{sc} = 2\% A_g = 0.02 A_g$

$A_c = A_g - A_{sc} = A_g - 0.02 A_g = 0.98 A_g$

$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$

$A_g = 104.16 \times 10^3 mm^2$

Assume rectangular column,  $A_g = b \cdot D$  Assume  $b = D/2$   
 $\therefore D = 456 \approx 500mm$   
 $b = 300mm.$

size of column = 500x300mm

check for eccentricity:-

$\frac{l_{eff}}{500} + \frac{b}{30} = 19.00 < 20mm$  Hence ok.

check for short/long column

$\lambda = \frac{l_{eff}}{b} = 15 > 12$  (Hence it is long column)

$A_{sc}$   $P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$   
 $A_{sc} = 3640 mm^2$

$x_{\text{steel}}, f = \frac{100 P_{sc}}{bD} = \frac{100 \times 3640}{800 \times 300} = 2.42\%$

No. of bars, Assume 25mm  $\Phi$

$n = \frac{A_{st}}{A \Phi} = 7.41 \approx 8 \text{ nos.}$

Provide 8 - 25mm  $\Phi$

Lateral ties

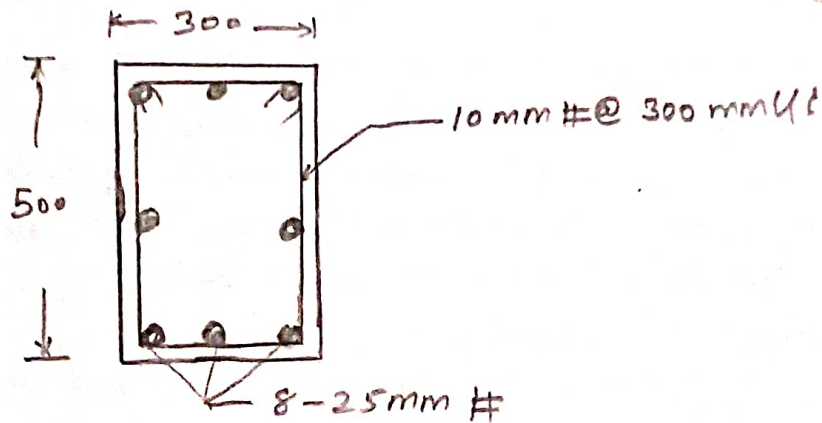
dia

- (i)  $2 \times \text{dia of main bar} / 4 = 6.25 \text{ mm}$
  - (ii) 5mm
- } Take greater value  
Take 10mm  $\Phi$

Spacing

- (i) least lateral dimensions = 300mm
  - (ii)  $16 \times \text{dia of small main bar} = 400$
  - (iii) 300mm
- } Least one  
 $s = 300 \text{ mm}$

Provide 10mm lateral ties @ 300mm  $\Phi$



2. A braced reinforced concrete column of circular section of 500mm dia is to support a factored axial load of 2300 kN along with a factored moment of 165 kN.m. The unsupported length of the column is 6.3m with eff. length of 5.5m. Design the column when it is to be provided with

- (i) lateral ties
- (ii) spiral reinforcement,

The M25 grade of concrete and HYSD steel bars of grade Fe 415.

Given data

$L_{\text{eff}} = 5.5 \text{ m} ; L = 6.3 \text{ m} ; D = 500 \text{ mm} ; P_u = 2300 \text{ kN} ;$   
 $M_u = 165 \text{ kN.m} ; f_{ck} = 25 \text{ N/mm}^2 ; f_y = 415 \text{ N/mm}^2 ;$  Assume  $d' = 40 \text{ mm}$

Non dimensional parameters

$\frac{d'}{D} = 0.1 ; \quad P_u / f_{ck} bD = 0.368$   
 $M_u / f_{ck} bD^2 = 0.053$

Main

From sp:16 ; chart 56 ;

$$\frac{P_u}{f_{ck} b d} = 0.368 \quad \& \quad \frac{M_u}{f_{ck} b d^2} = 0.053$$

$$\frac{p}{f_{ck}} = 0.06$$

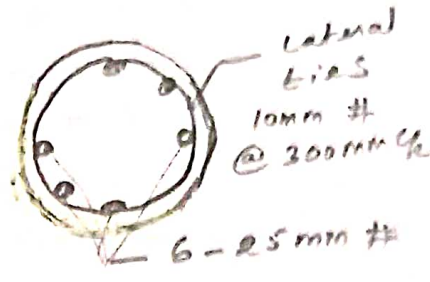
$$p = 1.5\%$$

$$A_{sc} = \frac{p \cdot \pi D^2}{100 \cdot 4} = 2945 \text{ mm}^2$$

Provide 6 - NOS 25 mm #

(i) Lateral ties

dia (i) Main dia = 6.25mm } Least one  
 (ii) 5mm } Provide 10mm #



Spacing

- (i) Least lateral dimensions = 500mm
- (ii) 16 x Main dia = 400mm
- (iii) 300mm

Provide 10mm lateral ties @ 300mm c/c.

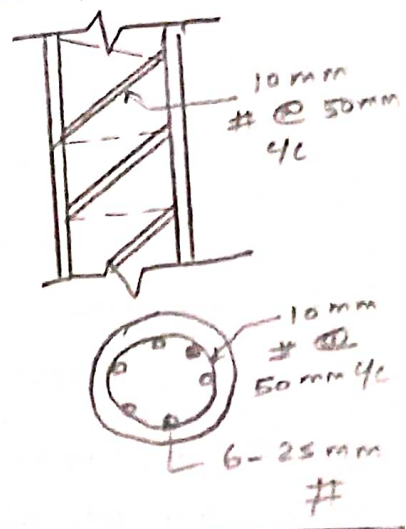
(ii) Helical reinforcement

(a) dia (i)  $\frac{1}{4}$  main bar = 6.25mm } Take 10mm #  
 (ii) less than 16mm

Spacing

- (i)  $\neq 75$ mm
- (ii)  $\neq \frac{1}{6} d_c = 66.67$ mm
- (iii)  $\neq 25$ mm
- (iv)  $\neq 3 \times \text{dia of spiral} = 3 \times 10 \neq 30$ mm.

Hence provide 10mm # @ 50mm c/c.



May/June 2016

1. Design the reinforcement in a circular column of dia 300mm to support a service axial load of 800kN. The column has an unsupported length of 3m and is braced against side sway. The column is reinforced with helical ties. The materials to be used are M25 grade of concrete & HYSD steel bars of grade Fe415.

Same as APR/MAY 2017 - Problem no: 1.

changes

$f_{ck}$  values.

2. Design the reinforcements in short column 400 x 400mm at the corner of a multistoried building to support an axial factored load of 1500kN, together with biaxial moments of 50kN.m acting in perpendicular planes. Adopt M20 grade of concrete and steel grade Fe415 HYSD bars.

Same as APR/MAY 2017 - Problem No: 2

changes

Nov/Dec 2015

1. Design a column having an eff. length of 4.75m to support a factored load of 1650kN. Consider the reinforcement ratio  $\rho$  to be in the range 1.5% to 2% and the eff. cover to longitudinal steel of 55mm. The materials to be used are M25 grade concrete & HYSD steel bars of grade Fe415.

Same as Nov/Dec 2016; Problem No: 1.

changes

\* Values

2. A braced reinforcement concrete column of circular cross section of 500mm diameter is to support a factored axial load of 2250kN along with a factored moment of 160kN.m. The unsupported length of the

column is 6.3 m, eff. length of 5.5 m. Design the column when it is to be provided with lateral ties and spiral reinforcement. The M25 grade of concrete & HYSD steel bars of grade Fe 415.

Same as Nov/Dec 2016 - problem NO: 2

changes:

\* Values.

April/May 2015

1. A rectangular reinforced concrete column of c/s dimensions 450 x 600 mm is subjected to an axial load of 200 kN under service dead load & live loads. The column has an unsupported length of 3 m. Adopt M20 grade concrete & Fe 415 steel. Design the column.

Given data

$L_0 = 3 \text{ m}$  ; size of column = 450 x 600 mm

$f_y = 415 \text{ N/mm}^2$  ;  $f_{ck} = 20 \text{ N/mm}^2$  ; axial load = 200 kN

Factored axial load = 300 kN.

$$\lambda = \frac{L}{D} = 6.67 < 12 \quad (\therefore \text{short column})$$

$$e_{\min} = \frac{L}{500} + \frac{D}{30} = 19.33 < 20 \quad (\text{eccentricity condition is satisfied})$$

$$\underline{A_{sc}} = P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

$$A_{sc} =$$

Lateral ties

(i)  $\frac{\text{Larger dia.}}{4}$

(ii) 5 mm

Spacing

(i) least lateral dimensions

(ii) 16 x dia of smallest member

(iii) 300 mm

## Unit - V

### Limit state Design of footing

#### Depth Requirements:

- \* depth should be adequate in oneway & two way shear without shear reinforcement.
- \* depth should be adequate
  - for bending moment without compression reinforcement
  - to transfer bond length by the main bars & dowel bars

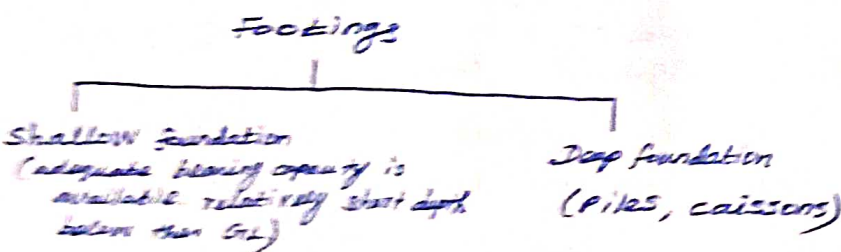
#### Reinforcement requirements:

- \* Minimum steel reinforcement, For Fe250 → 0.15%  $b \cdot d$   
For Fe415 → 0.12%  $b \cdot d$
- \* In case, the column bars are bent, they may be extended into the footing and such bars can also be considered to act with the footing.
- \* In sloped footing, check for minimum steel area needs to be made only at the middle of the footing.  
The max. spacing is  $3d$  (or) 300 mm

#### Footings

- \* Footings located below the ground level
- \* Purpose → to effectively support the superstructure like columns by transmitting the applied loads, moments & other forces to the soil.  
→ footings resist the BM & SF developed due to soil reaction.

#### Types of footing

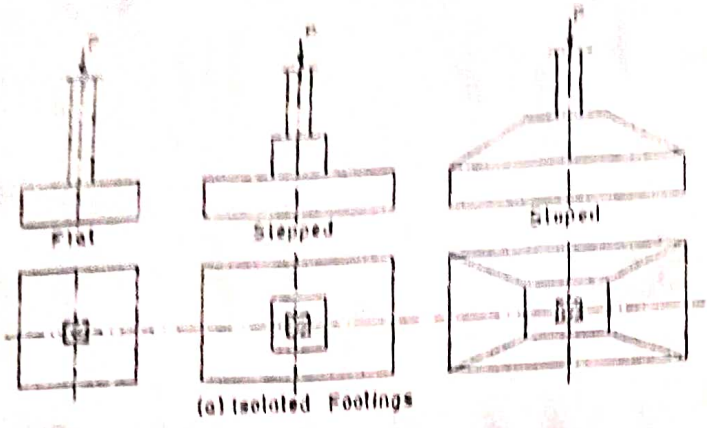


- Note Under working load
- \* safe bearing capacity of soil varies in the range of 100 to 400  $\text{kN/m}^2$  whereas
  - \* Comp. stress in concrete is around 5 to 15  $\text{N/mm}^2$  &
  - \* in steel 130 to 190  $\text{N/mm}^2$  in RC column (comp stress)

Types of footing — shallow footing — 
{

 Isolated column footing  
 Combined column footing

Isolated column footing :



- \* For framed building in firm soil → to provide separate independent footings for each column
- \* Types → square, Rectangular, circular
- \* Thick slab (base) → flat or stepped (or) sloped
- \* Footings are reinforced by a steel mesh located at bottom of the slab (to resist BM & SF developed due to soil pressure)

Combined Column Footing

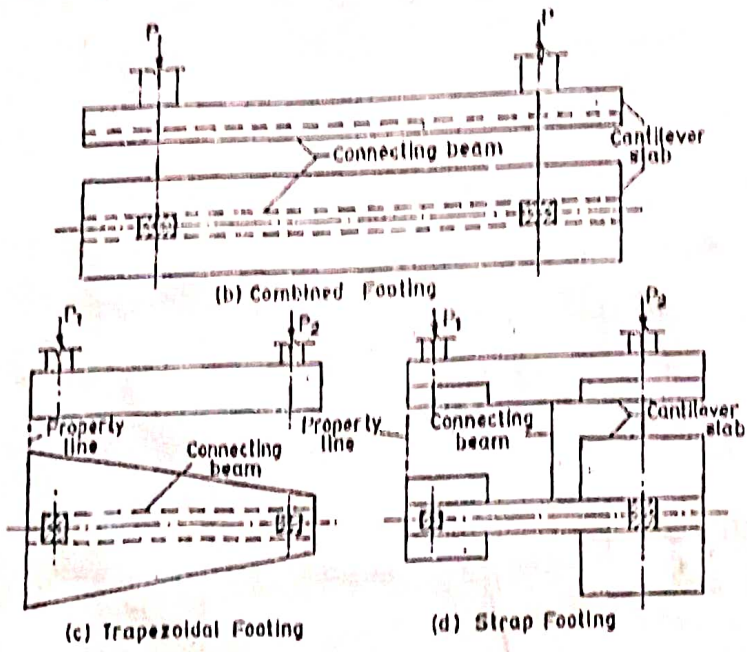


Fig. 10.21 Types of Footings

- \* Two or more heavy loaded columns are located close to each other resting on soil with low bearing capacity,
- \* The area of isolated footings overlap on each other
- \* columns located close to property line, footing cannot be extended on one side
- \* Combined footing — Rectangular Shape
- \* Combined footing → connecting beam b/w the columns integrally cast with a slab on either side of connecting beam.



Footings

- \* Column located close to the property line, footing cannot be extended on one side.
- \* Over come this problem, non availability of space near the exterior column, the footings of the interior & exterior columns are combined by using a connected beam & trapezoidal shaped slab.
- \* Due to the soil pressure, the slab bend transversely while the connecting beam bends longitudinally b/w the columns.

### \* Strap footing

- \* Alternative method of providing combined footing connecting column located on property line and the interior of the building
- \* In strap footing  $\rightarrow$  independent slabs are provided below the columns, connected by a strap beam.

### Design principles and codal requirements

- \* The design of the depth and reinforcements, is done for factored loads using the relevant safety factors applicable for limit state of collapse.
- \* The computations of factored moments & shear forces acting at the critical section of the footing is based on factored soil pressure corresponding to the factored loads on the column.
- \* Soil pressure developed due to the self weight of the footing does not induce any moments & SF & hence neglected in computations.
- \* Loads acting on the column & the soil pressure developed due to the service loads & factored soil pressure to be used in design.

### (a) General design features

- \* Footings are designed for flexure & shear, (both one way & two way action), bearing & bond. — mainly due to the soil pressure from the soffit (bottom) of the slab

\* The transfer of force from the column to the footing also safety against sliding & overturning  $\rightarrow$  horizontal force acting on the structure.

\* clear cover of footing = 75 mm

### (b) Depth of foundation

\* According to Rankine's theory,  
min. depth of foundation

$$h = \frac{q_s}{\gamma_s} \left( \frac{1 - \sin \phi}{1 + \sin \phi} \right)^2$$

safe bearing capacity of soil under the footing  
angle of repose  
unit wt of soil

### (c) Thickness of footing

\* Based on shear & flexure consideration - which are critical in the vicinity of the column & footing junction.

\* Minimum thickness = 150 mm

\* levelling course of lean concrete = 100 mm thickness  
(below the concrete)  
provided

### (d) Design for shear

\* shear force considerations - overall depth of footing.

\* check for shear stress, tension reinforcement in slab assumed 0.25 to 0.3% of

\* one way shear  $\rightarrow$  checked at critical section distant 'd' from the column face.

\* Two way shear (punching shear)  $\rightarrow$  similar to that of flat slab on columns.  
critical section ( $d/2$ ) from the periphery of the column.  
design ultimate SF

\*  $V_u$  is limited to shear resistance of concrete  $V_{uc}$  providing necessary depth.

\*  $V_u > V_{uc}$ , shear reinforcement required, to resist balance SF ( $V_u - V_{uc}$ )  
(similar to beam)

(e) Design for flexure

\* Larger concentration of reinforcement to be provided within a central band width equal to the width (shorter dimension) of the footing.

$$A_{st \text{ central band width}} = A_{st \text{ shorter}} \left[ \frac{2}{(\beta + 1)} \right]$$

$\beta \rightarrow$  ratio of long to short side of footing

(f) Force transfer at column base

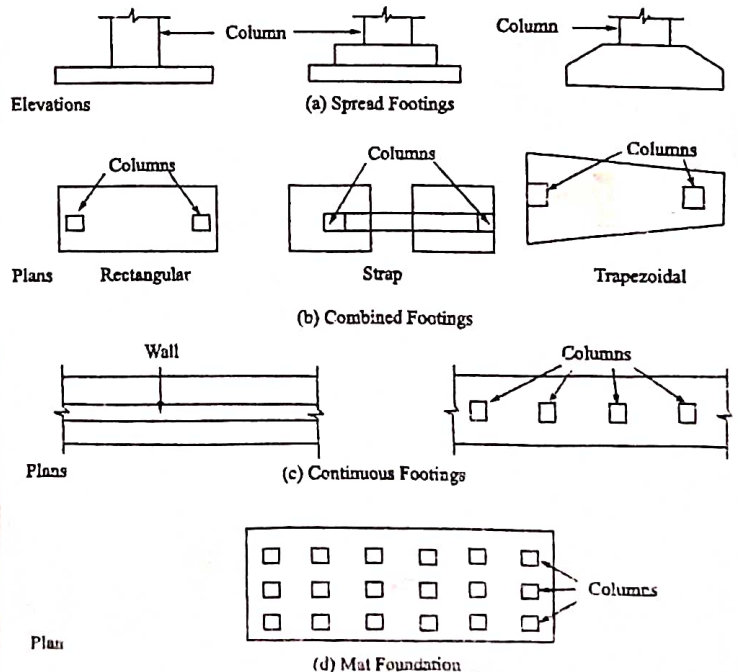
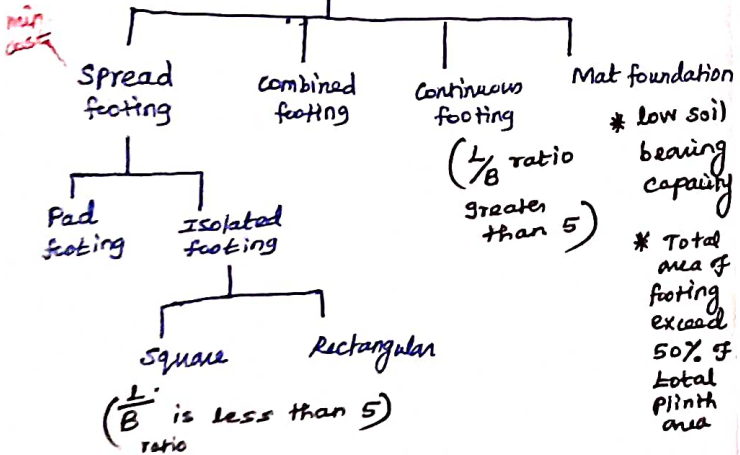
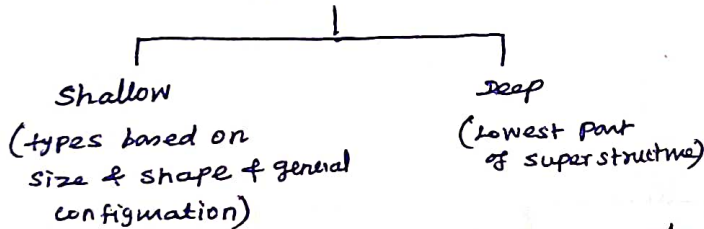
\* Axial force & moment acting at the base of the column must be transferred to the footing either by compression in concrete or by compression/tension in reinforcements.

\* Bearing resistance or comp. stress developed at the junction of column & footing is limited to a value by

$$f_{br \text{ max}} = 0.45 f_{ck} \sqrt{(A_1/A_2)}$$

$A_1 \rightarrow$  supporting area for bearing of footing (sloped/stepped)  
 $A_2 \rightarrow$  loaded area at the base of the column

Foundation



Load factor

- (i) D.L + L.L  $\rightarrow$  1.0 DL + 1.0 L.L
- (ii) D.L + Wind  $\rightarrow$  1.0 DL + 1.0 W.L
- (iii) D.L + imposed + Wind }  $\rightarrow$  1.2 DL + 0.8 LL + 0.8 WL (EQ.1)
- or EQ load }

Fig. 9.1. Types of shallow foundations

## Wall footing

1) Design a reinforced concrete footing for a 345 mm thick masonry wall which supports a characteristic load of 250 kN/m including self weight. Assume safe bearing capacity of soil is 150 kN/m<sup>2</sup> at a depth of 1.2 m below ground level. Assume M20 concrete & Fe415 steel used.

### Given data

safe bearing capacity of soil = 150 kN/m<sup>2</sup> ; characteristic load  $P = 250 \text{ kN/m}$   
Thickness of wall ( $t_w$ ) = 345 mm ; Depth below G.L = 1.2 m  
 $f_{ck} = 20 \text{ N/mm}^2$  ;  $f_y = 415 \text{ N/mm}^2$

### Step: 1 - Width/Breadth of footing

Assume self wt of footing & wt. of back fill  $\approx 10\%$  of load  
 $= \frac{10}{100} \times 250$   
 $= 25 \text{ kN/m}$

Total load (W) =  $250 + 25 = 275 \text{ kN/m}$

considered 1 m width of footing along the wall

$$\text{Area (A)} = \frac{\text{Total load}}{\text{S.B.C}}$$

$$A = b \cdot L \\ b = 1 \text{ m}$$

$$L \times 1 = \frac{275}{150} = 1.83 \text{ m}$$

Provide,  $L_f = 1.85 \text{ m}$

### Step: 2 - B.M. calculation

$$M = \frac{Wl^2}{2}$$

\* Projection from face of wall,

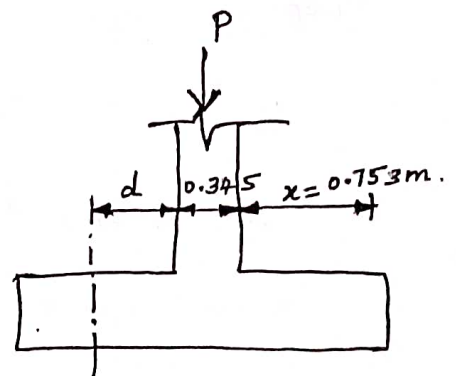
$$x = \frac{L_f - t_w}{2} = \frac{1.85 - 0.345}{2}$$

$$x = 0.753 \text{ m}$$

\* upward soil pressure,  $P_{upward} = \frac{\text{characteristic load (P)}}{A_{provided}}$

$$= \frac{250}{1.85 \times 1}$$

$$P_{upward} = 135 \text{ kN/m}^2$$



\* E.M @ face of wall

$$M = \frac{p x^2}{2} = \frac{125 \times 0.75^2}{2}$$

$$M = 28.22 \text{ kNm}$$

\* Factored moment,

$$M_u = 1.5 \times 28.22$$

$$M_u = 57.33 \text{ kNm}$$

$$\frac{x_{u, \max}}{d}$$

$$M_u = M_{u, \lim} = 67.33 \text{ kNm}$$

$$b = 1000 \text{ mm}$$

Step: 3 - Thickness/depth of footing

$$M_{u, \lim} = 0.36 \frac{x_{u, \max}}{d} \left(1 - 0.48 \frac{x_{u, \max}}{d}\right) b d^2 f_{ck}$$

$$d_{req} = \sqrt{\frac{M_u}{0.138 f_{ck} b}}$$

$$\therefore d_{req} = 144.13 \text{ mm}$$

Provide 16 mm # main reinforcement

$$D_{req} = d_{req} + \text{clear cover} + \frac{\phi}{2} = 144.13 + 50 + \frac{16}{2} = 202.13$$

$$D_{provided} = 2 \times D_{req}$$

↖ to avoid failure of footing in punching/two way shear

$$= 2 \times 202.13 = 404.26 \text{ mm}$$

Provide  $D = 405 \text{ mm}$

$$d = 405 - 50 - \frac{16}{2}$$

$$d = 347 \text{ mm}$$

$$b = 1000 \text{ mm}$$

$$d = 347 \text{ mm}$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

Step: 4 -  $A_{st}$

$$M_u = 0.87 f_y A_{st} \left(1 - \frac{f_y A_{st}}{f_{ck} b d}\right) d$$

$$A_{st} = 471.92 \text{ mm}^2$$

$$\text{spacing } (s) = \frac{1000 A_{st}}{A_{st}} = \frac{1000 \times \pi \times \frac{16^2}{4}}{471.92} = 425.9 \text{ mm c/c (16mm #)}$$

more spacing

$$(s) = \frac{1000 \times \pi \times \frac{12^2}{4}}{471.92} = 237.4 \text{ mm c/c (12mm # using)}$$

Provide 12 mm # @ 230 mm c/c

$A_{st, \min}$  - distribution reinforcement

$$A_{st, \min} = 0.12\% b D = \frac{0.12}{100} \times 1000 \times 405$$

$$A_{st, \min} = 486 \text{ mm}^2$$

Provide 12 mm # bars

$$\text{Spacing} = \frac{1000 A \phi}{A_{st}} = \frac{1000 \times \pi \times 12^2 / 4}{486} = 232.5 \text{ mm } \phi$$

Provide 12 mm # @ 230 mm  $\phi$

$$d = 405 - 50 = 355$$

$$d = 349 \text{ mm}$$

Step: 5 - check for one way/vertical shear

$$A_{o_{min}} = \frac{1000 A \phi}{S} = \frac{1000 \times \pi \times 12^2 / 4}{230} = 491.3 \text{ mm}^2$$

$$p_t = \frac{100 A_{st}}{bd} = \frac{100 \times 491.3}{1000 \times 349} = 0.14 \%$$

from IS: 456, 2000; Table: 19; PNO:

$$\tau_c = 0.28 \text{ N/mm}^2$$

\* Shear resisted by concrete,  $V_{uc} = \tau_c \cdot b \cdot d = 0.28 \times 1000 \times 349$

$$V_{uc} = 97.72 \text{ kN}$$

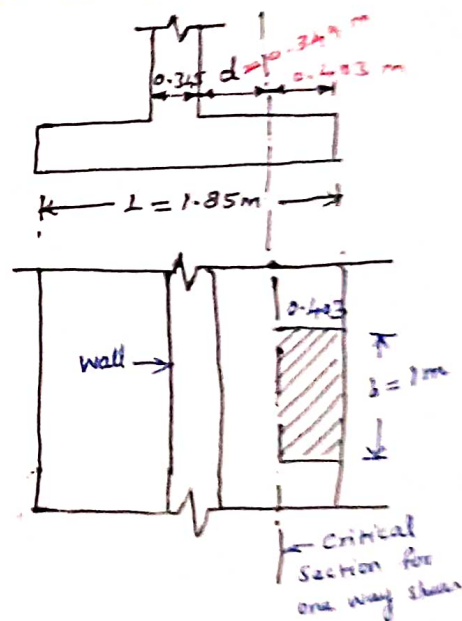
\* One way shear is critical at section 'd' from the face of the wall

$$V_u = 1.5 (p_{uprmd} \times \text{shaded area})$$

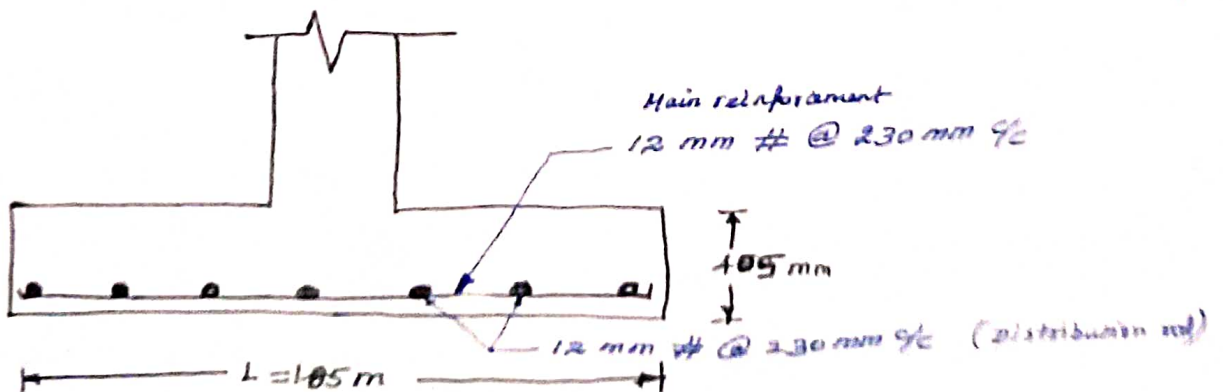
$$= 1.5 \times (135 \text{ kN/m}^2 \times 0.403 \times 1)$$

$$V_u = 81.61 \text{ kN} < 97.72 \text{ kN}$$

$V_u < V_{uc}$ , Hence safe in one way shear



Step: 6 - Reinforcement Details



## Rectangular footing

1) design a suitable footing for a RC column of size  $300 \times 500$  mm supporting a factored load of  $1500$  kN. Assume safe bearing capacity of soil as  $200 \text{ kN/m}^2$ . Adopt M20 concrete and Fe415 grades. sketch the details of reinforcement in footing. (May/June 2012)

Given data:

$$P_u = 1500 \text{ kN} ; q_s = 200 \text{ kN/m}^2 ; \text{Factored SBC} = 1.5 \times 200 = 300 \text{ kN/m}^2$$
$$b = 300 \text{ mm} ; D = 500 \text{ mm} ; f_{ck} = 20 \text{ N/mm}^2 ; f_y = 415 \text{ N/mm}^2$$

Step: 1 - Size of footing

\* Total factored load = Load on column + assume 10% self wt of footing

$$= 1500 + \left(\frac{10}{100} \times 1500\right)$$

$$W_u = 1650 \text{ kN}$$

\* Footing Area  $A_f = \frac{W_u}{\text{Factored SBC}} = \frac{1650}{300} = 5.5 \text{ m}^2 \approx 6 \text{ m}^2$

$$A_f = 6 \text{ m}^2$$

\* Proportion of footing area in the same proportion as the sides of the column.

$$\therefore 3x \times 5x = 6$$

$$15x^2 = 6$$

$$\therefore x = 0.63 \text{ m}$$

Size of column =  $300 \times 500$  mm  
proportion  $3x \times 5x$

$$\therefore \text{short side of footing} = 3x = 1.89 \text{ m} \approx 2 \text{ m}$$

$$\text{long side of footing} = 5x = 3.15 \text{ m} \approx 3 \text{ m}$$

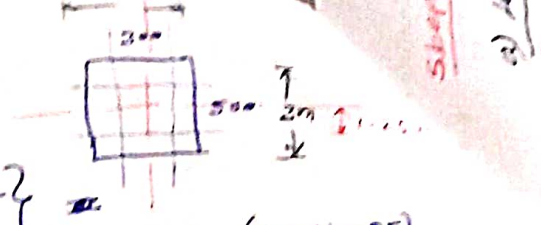
Adopt a rectangular footing of size  $2 \text{ m} \times 3 \text{ m}$

Check for footing Area

$$q_u = \frac{W_u}{A_f} = \frac{1650}{2 \times 3} = 275 \text{ kN/m}^2 < 300 \text{ kN/m}^2$$

Hence, the footing area is adequate.

Step: 2 - Factored B.M



\* Cantilever projection from the short face of the column } =  $\frac{3 - (0.25 + 0.25)}{2}$

$L_x = 1.25 \text{ m}$

\* Cantilever projection from the long face of the column } =  $\frac{3 - (0.15 + 0.15)}{2}$

$L_y = 0.85 \text{ m}$

$l_{short} = L_x$

\* B.M @ short side face of the column =  $\frac{q_u \cdot l_{short}^2}{2} = \frac{275 \times 1.25^2}{2}$

$M_x = 214.8 \text{ kN}\cdot\text{m}$   $l_{long} = L_y$

\* B.M @ long side face of the column =  $\frac{q_u \cdot l_{long}^2}{2} = \frac{275 \times 0.85^2}{2}$

$M_y = 99.34 \text{ kN}\cdot\text{m}$

Step: 3 - depth of footing

a) depth based on moment consideration IS: 456-2000  
P.No: 96, G.1.1(c)

$$M_u = 0.36 \frac{x_{u,max}}{d} \left[ 1 - 0.42 \frac{x_{u,max}}{d} \right] f_{ck} b d^2$$

$$M_u = 0.138 f_{ck} b d^2$$

$$\therefore d = \frac{M_u}{\sqrt{0.138 f_{ck} b}} = \frac{214.8 \times 10^6}{\sqrt{0.138 \times 20 \times 1000}}$$

$d = 279 \text{ mm}$

b) Depth based on shear stress (S.F per metre - longer direction)

$$V_{uL} = q_u \cdot [L_x - d] \text{ N}$$

$$= 275 [1250 - d] \text{ N}$$

IS: 456-2000  
P.No: 73 -  
table: 19

Assume, shear strength  $\tau_c = 0.36 \text{ N/mm}^2$  → For M20 grade concrete

Nominal % of steel,  $P_L = 0.25$

$$\tau_c = \frac{V_{uL}}{b d}$$

$$0.36 = \frac{275 [1250 - d]}{1000 \times d}$$

$\therefore d = 541 \text{ mm}$

Adopt  $d = 550 \text{ mm}$   $\phi$   $D = 600 \text{ mm}$



Step 4 - Reinforcement details

a) longer direction

IS:456-2000  
ANN: 16  
6.11(b)

$$M_{ux} = 0.87 f_y A_{st} d \left[ 1 - \frac{A_{st} \cdot f_y}{bd \cdot f_{ck}} \right]$$

$$A_{st1} = 1125.5 \text{ mm}^2$$

spacing,  $S = \frac{1000 A_d}{A_{st}}$

Adopt, 16 mm # @ 160 mm  $\phi$ .

(c) central band

Ratio of long to short side,  
 $\beta = \frac{L_y}{L_x} = \frac{3}{2} = 1.5$

Reinforcement in central band width of 2m

IS:456-2000,  
P.No: 65,  
cl: 34.3.1(c)

$$= \frac{2}{\beta + 1} \cdot A_{st}$$

$$= \frac{2}{1.5 + 1} \times (2 \times 565)$$

$$= 904 \text{ mm}^2$$

(b) shorter direction

$$M_{uy} = 0.87 f_y A_{st} d \left[ 1 - \frac{A_{st} \cdot f_y}{f_{ck} bd} \right]$$

$$A_{st_s} = 510 \text{ mm}^2$$

Adopt, 12 mm # @ 200 mm  $\phi$

d) A<sub>st</sub> min

For Fe415

$$A_{st_{min}} = 0.12\% \cdot bD$$

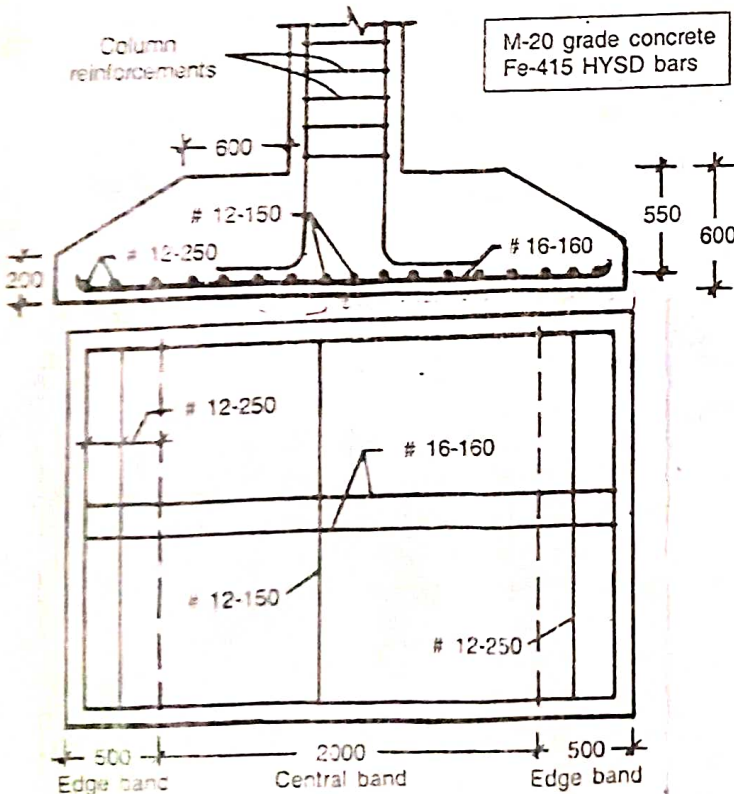
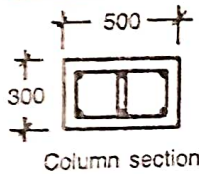
$$= \frac{0.12}{100} \times 1000 \times 600$$

$$= 2 \times 720$$

$$= 1440 \text{ mm}^2 > 904 \text{ mm}^2$$

provide 12 mm # @ 150 mm  $\phi$

Step 5 - Reinforcement details



Step 5 - Check for Shear Stress

The critical section for one way shear is located at 'd' from the face of column

$$\text{Ultimate S.F/m width in longer direction } (V_u) = \frac{275(L_x - d)}{1000} = \frac{275(1250 - 550)}{1000} = 275 \times 0.7 = 192.5 \text{ KN}$$

IS:456-2000; P.No: 73, Table: 19

$$p = \frac{100 A_{st}}{bd} = \frac{100 \times 1257}{1000 \times 550} = 0.228$$

From IS:456-2000, Table: 19

$$\begin{aligned} \text{* Permissible shear stress} &= k_s \cdot \tau_c \\ &= 1 \times 0.343 \\ &= 0.343 \text{ N/mm}^2 \end{aligned}$$

$$\begin{aligned} \text{* Nominal Shear stress, } \tau_v &= \frac{V_u}{bd} \\ &= \frac{192.5 \times 10^3}{1000 \times 550} \\ &= 0.35 \text{ N/mm}^2 \end{aligned}$$

The stress is almost equal, hence satisfactory, otherwise  $A_{st}$  (or)  $d$  may be increased.

## Wall footing

- 2) Design a footing for 250 mm thick wall which supports a load of 200 kN/m at service state for the following data:
- safe bearing capacity of soil ( $q_s$ ) — 150 kN/m<sup>2</sup>  
Angle of repose of soil ( $\phi$ ) — 30°  
unit wt of soil ( $\gamma_w$ ) — 20 kN/m<sup>3</sup>

### Step 1 - depth of foundation

$$h = \frac{q_s}{\gamma_w} \cdot \left( \frac{1 - \sin \phi}{1 + \sin \phi} \right)^2 = \frac{150}{20} \left( \frac{1 - \sin 30}{1 + \sin 30} \right)^2$$

$$h_{\text{found}} = 0.83 \text{ m}$$

Provide <sup>1m</sup> minimum depth of footing,  $\boxed{h_{\text{found}} = 1 \text{ m}}$

### Step 2 - width of footing

Factored load,  $W_u = 1.5 \times 200 = 300 \text{ kN/m}$

Factored Soil pressure  $q_u = 1.5 \times 150 = 225 \text{ kN/m}^2$

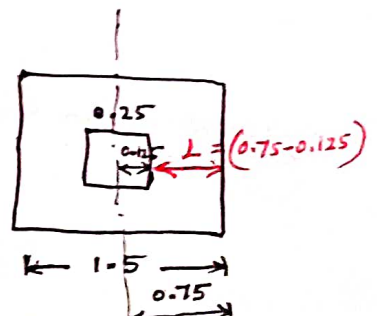
Considering 1 m length of footing.

$$B \times 1 = \frac{\text{load}}{q_u} = \frac{300}{225}$$

$$\therefore B = 1.33 \text{ m}$$

Provide,  $\boxed{B_f = 1.5 \text{ m}}$

$$A_f = \frac{W_u}{q_u}$$



### Step 3 - depth of footing

$$\text{B.M.}, M = \frac{q_u \cdot L^2}{2} = \frac{225 \times (0.75 - 0.125)^2}{2}$$

$$M = 43.95 \text{ kN.m}$$

IS: 456-2000  
P. No. 196, Cl. A. 11.1(c)

$$M_u = 0.36 \frac{x_{\text{max}}}{d} \left[ 1 - 0.42 \frac{x_{\text{max}}}{d} \right] f_{ck} b d^2 \quad (\text{or}) \quad M_u = 0.138 f_{ck} b d^2$$

$$d = \sqrt{\frac{43.95 \times 10^6}{0.138 \times 20 \times 1000}} = 126 \text{ mm}$$

$$\boxed{d = 150 \text{ mm}}$$

$$\boxed{D = 200 \text{ mm}}$$

step 4

step: 4 - Reinforcement

$$M_u = 0.87 f_y A_{st} \cdot d \left[ 1 - \frac{f_y A_{st}}{f_{ck} b d} \right]$$

$M_u = 43.95 \text{ kNm}$   
 $f_y = 415 \text{ N/mm}^2$   
 $f_{ck} = 20 \text{ N/mm}^2$   
 $b = 1000 \text{ mm}$   
 $d = 150 \text{ mm}$

$$A_{st} = 931.6 \text{ mm}^2$$

Assume 16 mm #

$$\text{spacing (s)} = \frac{1000 A_{\phi}}{A_{st}} = \frac{1000 \times \pi \times 16^2 / 4}{931.6} = 215 \text{ mm}$$

Provide 16 mm # @ 200 mm c/c

distribution Reinforcement

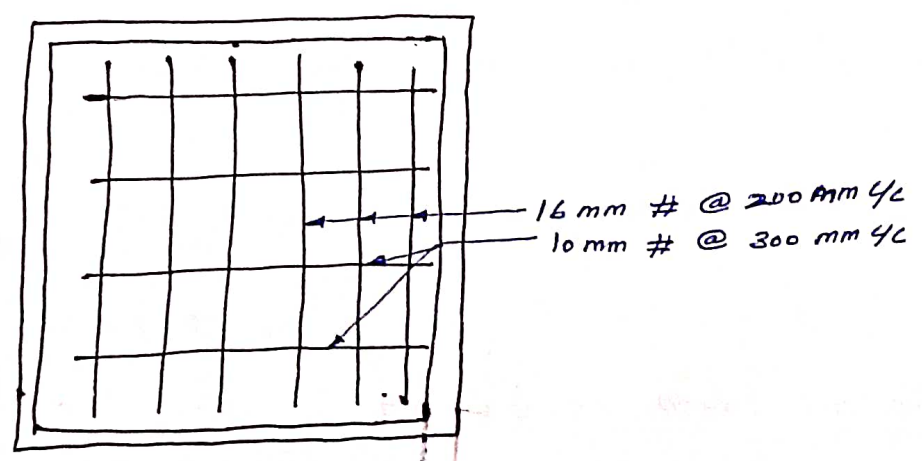
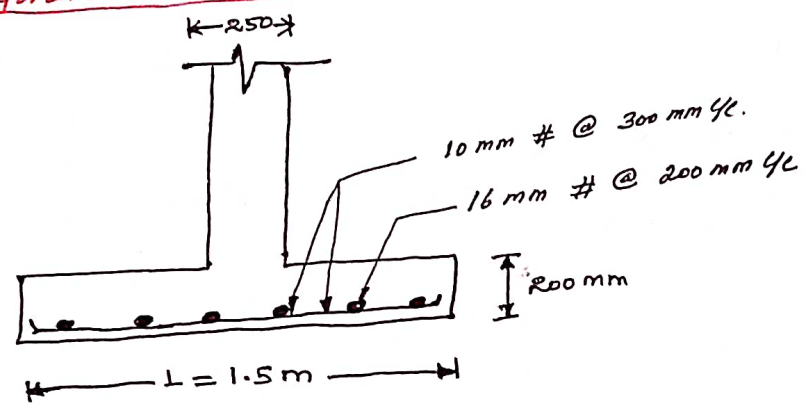
$$A_{st \text{ min}} = 0.12\% b D = \frac{0.12}{100} \times 1000 \times 200 = 240 \text{ mm}^2$$

Assume 10 mm #

$$s = \frac{1000 \times \pi \times 10^2 / 4}{240} = 327 \text{ mm}$$

Provide 10 mm # @ 300 mm c/c.

step: 5 - Reinforcement details



- 3) A square footing has to transfer a dead load of 900 kN and an imposed load of 500 kN for a square column of 400 mm. Assume the safe bearing capacity of the soil as 200 kN/m<sup>2</sup>. Design a square footing to support the above column. Adopt M20 grade concrete & Fe 415 grade steel.

Given data

$D.L = 900 \text{ kN}$  ;  $f_{ck} = 20 \text{ N/mm}^2$  ;  $b = D = 400 \text{ mm}$   
 $L.L = 500 \text{ kN}$  ;  $f_y = 415 \text{ N/mm}^2$  ;  $q_u = 200 \text{ kN/m}^2$   
 Factored SBC of soil  $q_u = 1.5 \times 200 = 300 \text{ kN/m}^2$

Step: 1 - size of footing

\* Total factored load ( $W_u$ ) = D.L + L.L = 1400 kN

\* Footing Area  $A_f = \frac{W_u}{q_u} = \frac{1400}{300} = 4.7 \text{ m}^2$

\* Side of square footing,  $= \sqrt{4.7} = 2.17 \text{ m}$

Adopt a square footing of size 2.30 m x 2.30 m.

\* check for footing Area

Factored soil pressure at base  $= \frac{W_u}{A_f} = \frac{1400}{(2.3)^2} = 264.7 \text{ kN/m}^2 < 300 \text{ kN/m}^2$   
 Hence footing Area is adequate.

Step: 2 - Factored B.M

\* Cantilever projection from the face of the column  $= \frac{2.3 - 0.4}{2} = 0.95 \text{ m}$

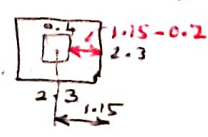
\* B.M on either side  $M_u = \frac{q_u \cdot L_c^2}{2} = \frac{264.7 \times 0.95^2}{2} = 120 \text{ kNm}$

Step: 3 - Depth of footing

a) depth based on moment consideration

$M_u = 0.36 \frac{x_{u,max}}{d} \left(1 - 0.42 \frac{x_{u,max}}{d}\right) f_{ck} b d^2$  (or)  $0.138 f_{ck} b d^2$

$d = \sqrt{\frac{120 \times 10^6}{0.138 \times 20 \times 1000}} = 208.51 \text{ mm}$



b) based on shear consideration

S.F per metre width,  $V_u = q_u (L_x - d)$  Newton

$V_u = 264.7 (0.95 - d)$  Newton

IS: 456-2000: P. No: 73, table: 19

Assume,  $\tau_c = 0.36 \text{ N/mm}^2$ , for M20 grade concrete  
Nominal % of steel  $P_E = 0.25$

$$\tau_c = \frac{V_u}{bd}$$

$$0.36 = \frac{264.7 (950 - d)}{1000 d}$$

$$d = 402.5 \text{ mm}$$

Provide  $d = 450 \text{ mm}$ :

$$D = 500 \text{ mm}$$

### Step: 4 - Reinforcement details

$$M_u = 0.87 f_y A_{st} d \left[ 1 - \frac{f_y A_{st}}{f_{ck} b d} \right]$$

$$A_{st} = 868 \text{ mm}^2$$

Assume 16 mm #

$$\text{spacing } (s) = \frac{1000 A_{st}}{A_{st}} = \frac{1000 \times \pi \times 16^2 / 4}{868} = 200 \text{ mm c/c}$$

Provide 16 mm # @ 200 mm c/c. ( $A_{st} = 1004.8 \text{ mm}^2$ )

$M_u = 120 \text{ kN}\cdot\text{m}$   
 $f_y = 415 \text{ N/mm}^2$   
 $f_{ck} = 20 \text{ N/mm}^2$   
 $b = 1000 \text{ mm}$   
 $d = 450 \text{ mm}$

### Step: 5 - check for shear stress

\*  $V_u = 264.7 (L_x - d) = 264.7 (950 - 450)$

$$V_u = 132.4 \text{ kN}$$

IS: 456-2000  
P. No: 73  
Table: 19

$$p = \frac{100 A_{st}}{bd} = \frac{100 \times 1004.8}{1000 \times 450} = 0.223$$

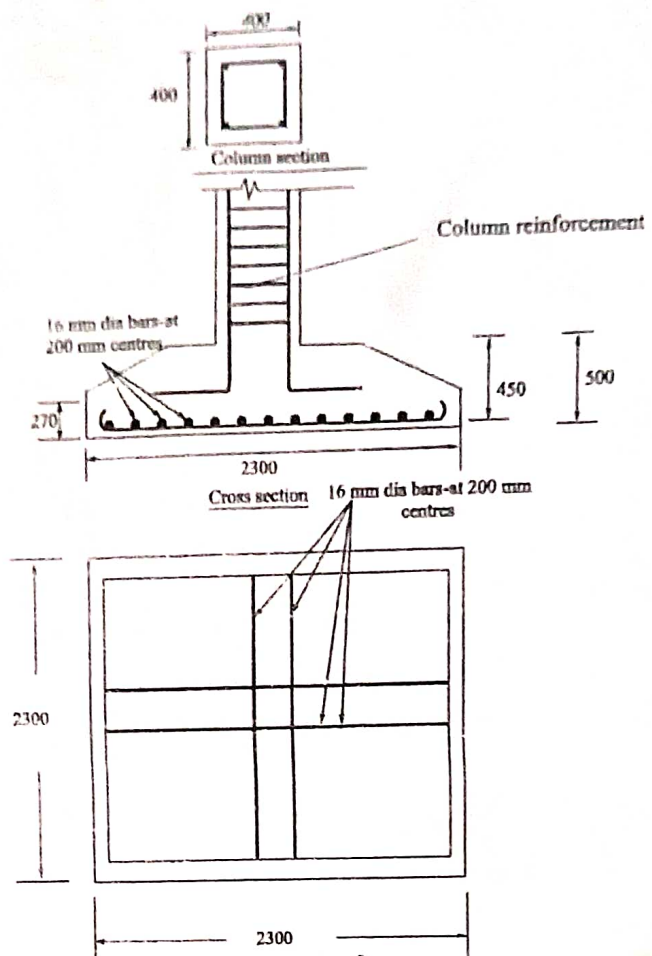
\* Permissible shear stress  $= k_s \cdot \tau_c = 1 \times 0.334 = 0.334 \text{ N/mm}^2$

\* Nominal shear stress  $\tau_v = \frac{V_u}{bd} = \frac{132.4 \times 10^3}{1000 \times 450} = 0.294 \text{ N/mm}^2$

$$\tau_v < k_s \cdot \tau_c$$

∴ Shear stresses within the safe permissible limits

Step: 6 - Reinforcement details



## Combined Rectangular footing

- 1) design a combined footing for the two column at a multistorey building. The columns of size 400 mm x 400 mm transmit a working load of 500 kN each and they are spaced at 5 m centres. The safe bearing capacity of the soil at site is 200 kN/m<sup>2</sup>. Adopt M20 grade concrete & Fe415 grade steel. Sketch the details of reinforcement in the combined footing.

### Given Data

size of column = 400 x 400 mm

Spacing of column = 5 m

Working load on each column = 500 kN

Bearing capacity of soil  $q_s = 200 \text{ kN/m}^2$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

### Step: 1 - Loads on footing

Factored load on each column =  $1.5 \times 500 = 750 \text{ kN}$

$\therefore$  Total load on column =  $2 \times 750 = 1500 \text{ kN}$

Self wt (10% assumed) = 150 kN.

$\therefore$  Total factored load  $P_u = 1650 \text{ kN}$ .

### Step: 2 - size of footing

Factored bearing capacity =  $1.5 \times 200 = 300 \text{ kN/m}^2$

Area of footing =  $\frac{\text{Total factored load}}{\text{Factored bearing capacity}}$

$$A_f = \frac{P_u}{SBC_{\text{fact}}} = \frac{1650}{300} = 5.5 \text{ m}^2$$

$$A_f = 5.5 \text{ m}^2$$

spacing b/w columns = 5 m each side 1 m (5+1+1=7m)

\* Adopt a footing size of 7 m x 1.5 m ( $A = 10.5 \text{ m}^2$ ) giving projection from column about 500 mm.

\* Assumed combined footing with a strap beam.

\* Adopt width of strap beam =  $400 + 50 + 50 = 500 \text{ mm}$

### Step: 3 - Design of footing

\* Soil pressure  $q_u = \frac{P_u^{\text{Total load}}}{A_f} = \frac{1650}{7 \times 1.5} = 157 \text{ kN/m}^2 < 300 \text{ kN/m}^2$

\* Cantilever projection of footing (L) =  $\left( \frac{1.5 - 0.5}{2} \right) = 0.5 \text{ m}$ .

\* Ultimate design moment,  $M_u = \frac{q_u \cdot L^2}{2} = \frac{157 \times 0.5^2}{2}$

$$M_u = 19.6 \text{ kN.m}$$

\* Effective depth of footing

$$d = \sqrt{\frac{M_u}{0.138 f_{ck} \cdot b}} = \sqrt{\frac{19.6 \times 10^6}{0.138 \times 20 \times 1000}} = 84.3 \text{ mm}$$

\* Depth based on shear consideration will be nearly double than that ~~required~~ due to moment considerations.

$\therefore$  Adopt effective depth ( $d$ ) = 250 mm

overall depth ( $D$ ) = 300 mm

\* Area of reinforcement

$$M_u = 0.87 f_y A_{st} \cdot d \left(1 - \frac{A_{st} \cdot f_y}{b d f_{ck}}\right)$$

$$\begin{aligned} f_y &= 415 \text{ N/mm}^2 \\ f_{ck} &= 20 \text{ N/mm}^2 \\ b &= 1000 \text{ mm} \\ d &= 250 \text{ mm} \end{aligned}$$

$$A_{st} = 222 \text{ mm}^2$$

$$A_{st \text{ min}} = 0.12\% \cdot bD = \frac{0.12}{100} \times 1000 \times 300 = 360 \text{ mm}^2$$

Assume 10 mm # bar

$$\text{Spacing (s)} = \frac{1000 A_{st}}{A_{st}} = \frac{1000 \times \pi \cdot 10^2 / 4}{360} = 218$$

Main reinforcement

Provide 10 mm # @ 200 mm c/c. ( $A_{st} = 392.7 \text{ mm}^2$ )

Step: 4 - check for shear stress

\* Shear stress at a distance equal to the eff. depth is

$$V_u = (L - d) q_u = (0.5 - 0.25) \times 157$$

$$V_u = 39.3 \text{ kN}$$

$$\tau_v = \frac{V_u}{b d} = \frac{39.3 \times 10^3}{1000 \times 250} = 0.158 \text{ N/mm}^2$$

$$p = \frac{100 A_{st}}{b d} = \frac{100 \times 392.7}{1000 \times 250} = 0.157$$

From IS 456:2000 ; Table : 19 ,  $\tau_c = 0.28 \text{ N/mm}^2$

\* Permissible shear stress =  $k_s \tau_c = 1 \times 0.28 = 0.28 \text{ N/mm}^2$

Since  $k_s \cdot \tau_c > \tau_v$ ,

Shear stress within safe permissible limits.



## Step 5 - Design of Strip beam

\* Factored load on the beam  $W_u = 1.5 \times 157 = 236 \text{ kN/m}$

Neglecting the small cantilever projection of beam

$$M_u = \frac{W_u L^2}{8} = \frac{236 \times 5^2}{8} = 738 \text{ kNm}$$

$$V_u = \frac{W_u L}{2} = \frac{236 \times 5}{2} = 590 \text{ kN}$$

\* Depth of strip beam is computed based on the moment which will be less than shear consideration.

Assuming  $\tau_c = 1.8 \text{ N/mm}^2$

$$\tau_c = \frac{V_u}{bd}$$

$$\therefore d = \frac{V_u}{b \cdot \tau_c} = \frac{590 \times 10^3}{500 \times 1.2} = 983 \text{ mm} \approx 1000 \text{ mm}$$

eff. depth,  $d = 1000 \text{ mm}$

overall depth  $D = 1150 \text{ mm}$

\*  $A_{st}$

$$M_u = 0.87 f_y A_{st} d \left[ 1 - \frac{A_{st} f_y}{f_{ck} bd} \right]$$

$$\boxed{A_{st} = 2257 \text{ mm}^2}$$

Provide 5 nos of 25 mm  $\phi$  ( $A_{st} = 2453 \text{ mm}^2$ )

$$\begin{aligned} M_u &= 738 \times 10^6 \text{ Nmm} \\ d &= 1000 \text{ mm} \\ b &= 500 \text{ mm} \\ f_y &= 415 \text{ N/mm}^2 \\ f_{ck} &= 20 \text{ N/mm}^2 \end{aligned}$$

\* Shear Stress,  $\tau_v = \frac{V_u}{bd} = \frac{590 \times 10^3}{500 \times 1000} = 1.18 \text{ N/mm}^2$

$$\phi = \frac{100 A_{st}}{bd} = \frac{100 \times 2453}{500 \times 1000} = 0.49\%$$

IS: 456-2000; Table: 19

Permissible shear stress,  $\tau_c = 0.46 \text{ N/mm}^2 < \tau_v$

Shear reinforcement are required, to resist the balanced shear force, with

$$V_{u1} = (V_u - \tau_c \cdot b \cdot d)$$

$$= (590 \times 10^3 - 0.46 \times 500 \times 1000)$$

$$V_{u1} = 360 \text{ kN}$$

using 10mm # 4 legged stirrups,

$$Spacing S_r = \frac{0.87 \frac{f_y}{\gamma_u} \cdot A_{st} \cdot d}{V_u} = \frac{0.87 \times 415 \times \left( \frac{A_{st}}{s} \times 1150 \right) \times 1150}{360 \times 10^3}$$

$$= 316 \text{ mm}$$

Adopt, 10mm # 4 legged stirrups @ 300 mm c/c. - stirrup beam

\* side face reinforcement = 0.1% of web area (as per IS 456-2000)

Step: 6 - Reinforcement Details

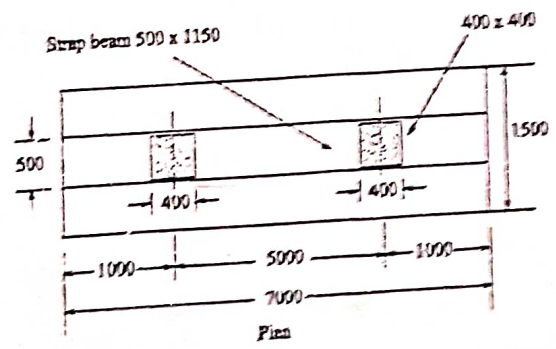
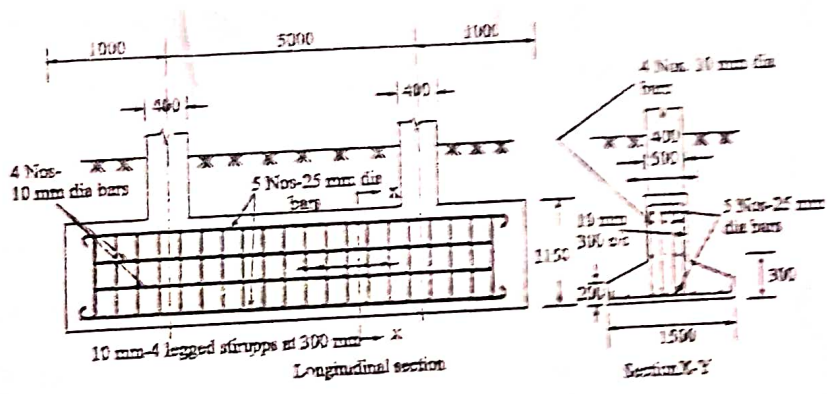


Fig. 9.11 Detailed reinforcement of combined footing

Apr/May 2017

1. A rectangular RCC column of size 400 mm x 600 mm carrying an axial load of 1800 kN. If the safe bearing capacity of the soil is 150 kN/m<sup>2</sup>. Design a suitable footing. Use M20 concrete & Fe 415 steel.

Given data

$$P_u = 1800 \text{ kN} ; b = 400 \text{ mm} ; D = 600 \text{ mm} ; f_{ck} = 20 \text{ N/mm}^2$$
$$f_y = 415 \text{ N/mm}^2 ; q_s = 150 \text{ kN/m}^2 ; \therefore q_u = 1.5 q_s = 225 \text{ kN/m}^2$$

Size of footing

$$\text{Total load incl. self wt} = 10\% P_u + P_u = 1980 \text{ kN} \approx 2000 \text{ kN}$$

$$\text{Footing Area} = \frac{\text{Total load}}{q_u} = \frac{2000}{225} = 8.9 \approx 10 \text{ m}^2$$

Proportion

$$4x \times 6x = 10$$

$$24x^2 = 10$$

$$\therefore x = 0.65$$

$$\text{Short side of footing} = 4x = 2.63 \text{ m} \approx 2.5 \text{ m}$$

$$\text{Long side of footing} = 6x = 3.94 \text{ m} \approx 4 \text{ m}$$

$$\text{Factored soil pressure } q_u = \frac{\text{Tot. load} = 2000}{\text{Area of footing } 4 \times 2.5} = 200 \text{ kN/m}^2$$

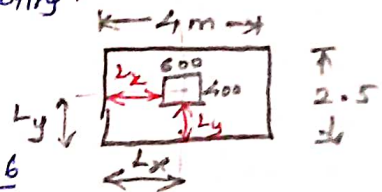
Factored B.M

Cantilever projection, Long side =  $\frac{4 - 0.6}{2} = 1.7 \text{ m}$

$$\text{Short side} = \frac{2.5 - 0.4}{2} = 1.05 \text{ m}$$

$$\text{Short side B.M} = \frac{q_u \cdot L_y^2}{2} = 110.25 \text{ kN.m}$$

$$\text{Longer side B.M} = \frac{q_u \cdot L_x^2}{2} = 289 \text{ kN.m}$$



Depth of footing

$$a) M_u = 0.138 f_{ck} b d^2 ; d = 323.6 \text{ mm}$$

$$b) \text{Shear consideration, } V_{uL} = q_u \left( \frac{L_y}{2} - \frac{600}{2} - d \right) N$$
$$= 200 (1700 - d) N$$

Assume  $\tau_c = 0.36 \text{ N/mm}^2$  for M20 concrete  
 $p = 0.25$

$$\tau_c = \frac{V_u L}{bd} \quad \therefore d = 250 \text{ mm.}$$

Adopt  $d = 350 \text{ mm}$  &  $D = 400 \text{ mm}$

### Reinforcement

#### Longer direction

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{f_y A_{st}}{f_{ck} bd}\right)$$

$$A_{st} = 2728 \text{ mm}^2$$

Assume dia of bar = 20 mm

$$\text{Spacing} = \frac{1000 A_{\phi}}{A_{st}} = 115 \text{ mm} \approx 100 \text{ mm}$$

Provide 20 mm # @ 100 mm c/c

#### Shorter direction

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{f_y A_{st}}{f_{ck} bd}\right)$$

$$A_{st} = 920.7 \text{ mm}^2$$

Assume 16 mm #

$$\text{Spacing} = \frac{1000 A_{\phi}}{A_{st}} = 200 \text{ mm}$$

Provide 16 mm # @ 200 mm c/c

### Central Band

Band width = width of footing = 2.5 m

$$\frac{\text{Reinforcement in central Band}}{\text{Total reinforcement in Short direction}} = \frac{2}{\beta + 1} \quad ; \quad \beta = \frac{\text{Longer side}}{\text{Shorter side}}$$

$$\beta = 1.6$$

$$\text{Reinforcement in Central Band of 2.5 m} = \left(\frac{2}{\beta + 1}\right) A_{st_{\text{Short}}} \times b_f$$

$$A_{st_{cb}} = 1770.6 \text{ mm}^2$$

$$A_{st_{\text{min}}} = 0.12 \% bD = 960 \text{ mm}^2 < 1770.6 \text{ mm}^2$$

Hence provide 16 mm # @ 110 mm c/c.

### Check for Shear stress

$$\text{Longer direction } V_{uL} = q_u (L_y - d) = 270 \text{ kN.}$$

$$\frac{100 A_{stL}}{bd} = 0.9$$

IS: 456-2000; P-NO: 73; Table: 19

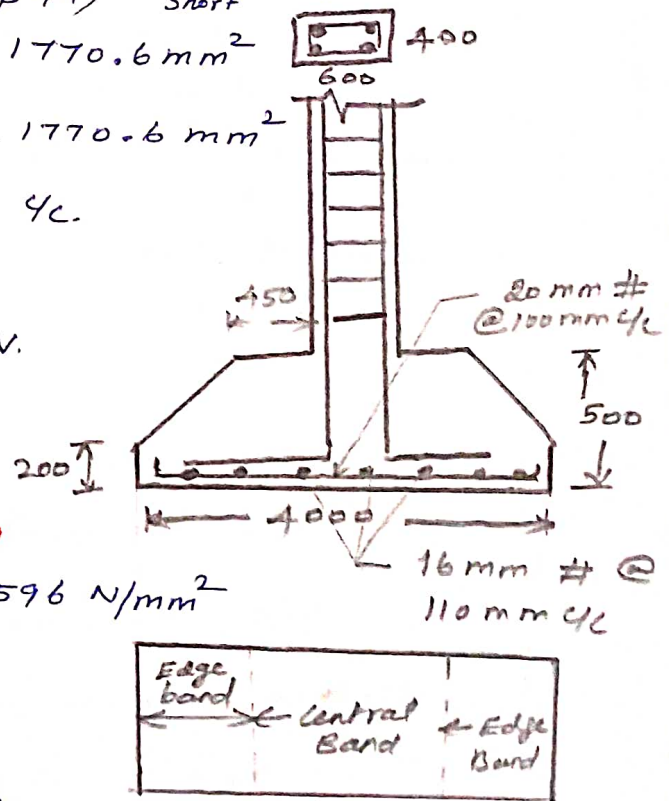
$$k_s \cdot \tau_c = 1 \times 0.596 = 0.596 \text{ N/mm}^2$$

$$\tau_v = \frac{V_{uL}}{bd} = 0.77 \text{ N/mm}^2$$

$$\text{check } k_s \cdot \tau_c = \tau_v$$

$$\therefore \tau_v = 0.596 = \frac{V_u}{bd} \quad ; \quad d = 453 \text{ mm.}$$

$\therefore$  revised depth of 450 mm &  $D = 500 \text{ mm.}$



design a suitable footing for a 500 mm x 500 mm square column transferring 100 kN axial load and a moment of 35 kNm. The safe bearing capacity of soil is  $190 \text{ kN/m}^2$ . Use M20 concrete and Fe415 steel. Adopt limit state design method.

Same as previous problem procedure

Changes

Square footing ( $\therefore$  shorter direction = longer direction)

Nov/dec 2016

1. A 230 mm thick masonry wall is to be provided with a reinforced concrete footing on a site having soil with SBC, unit weight and angle of repose of  $130 \text{ kN/m}^2$ ,  $17.5 \text{ kN/m}^3$  and  $30^\circ$  respectively. The M20 grade of concrete and HYSD steel bars of grade Fe415. Design the footing when the wall supports at service state: a load of  $150 \text{ kN/m}$  length.

Given data

Wall thickness = 230 mm ; service load =  $150 \text{ kN/m}$   
 unit wt of soil ( $\gamma_w$ ) =  $17.5 \text{ kN/m}^3$  ;  $\therefore$  factored load ( $W_u$ ) =  $225 \text{ kN/m}$   
 SBC  $q_s = 130 \text{ kN/m}^2$  ;  $f_{ck} = 20 \text{ N/mm}^2$  &  $f_y = 415 \text{ N/mm}^2$   
 $\phi = 30^\circ$

Depth of foundation  $h = \frac{q_s}{\gamma_w} \left( \frac{1 - \sin \phi}{1 + \sin \phi} \right)^2 = 0.83 \text{ m} \approx 1 \text{ m}$ .  
 (min. depth of foundation)

Width of footing

Factored soil pressure,  $q_u = 1.5 q_s = 195 \text{ kN/m}^2$

considering 1m length of footing

$q_u = \frac{\text{load}}{\text{Area}} = \frac{W_u}{B \times L}$  ; Width of footing =  $1.15 \text{ m} \approx 1.50 \text{ m}$ .

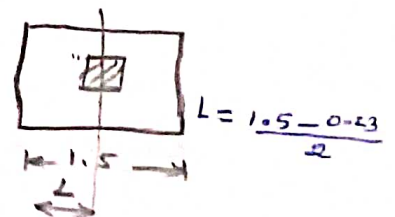
Depth of footing

$$B.M = \frac{q_u \cdot L^2}{2} = 39.31 \text{ kN.m}$$

$$M_u = 0.138 f_{ck} b d^2$$

$$\therefore d = 119 \text{ mm} \approx 150 \text{ mm}$$

$$D = 200 \text{ mm}$$



## Reinforcement

$$M_u = 0.87 f_y A_{st} d \left( 1 - \frac{f_y A_{st}}{f_{ck} b d} \right)$$

$$A_{st} = 840 \text{ mm}^2$$

No. of bar  
spacing

2. A Rectangular column  $550 \times 350 \text{ mm}$  carries a load of  $775 \text{ kN}$ . Design a rectangular footing to support the column. The safe bearing capacity of the soil is  $210 \text{ kN/m}^2$ . Use M15 concrete.

Same as Apr/May 2017 - Problem No: 1 Procedure

changes

Assume, Fe 415 or Fe 250

## May/June 2016

1. Design a reinforced concrete footing for a rectangular column of section  $300 \text{ mm} \times 500 \text{ mm}$  supporting an axial factored load of  $1500 \text{ kN}$ . The safe bearing capacity of the soil at site is  $185 \text{ kN/m}^2$ . Adopt M20 grade of concrete and HYSD steel bars of grade Fe 415.

Same as Apr/May 2017 - Problem No: 1 Procedure.

2. Design a combined column footing with a strap beam for two reinforced concrete columns  $300 \times 300 \text{ mm}$  size spaced  $4 \text{ m}$  apart and each supporting a factored axial load of  $750 \text{ kN}$ . Assume the ultimate bearing capacity of soil at site as  $225 \text{ kN/m}^2$ . Adopt M20 grade of concrete & steel grade Fe 415 HYSD bars.

### Given data

Size of column =  $300 \times 300 \text{ mm}$  ; Spacing of column =  $4 \text{ m}$

Factored load on each column =  $750 \text{ kN}$  ;  $\therefore$  Total load =  $1500 \text{ kN}$

Ultimate SBC of soil =  $225 \text{ kN/m}^2$  ;  $f_{ck} = 20 \text{ N/mm}^2$  ;  $f_y = 415 \text{ N/mm}^2$

### Loads on footing

$$W = (750 + 750) + 10\% \text{ extra self wt} = 1650 \text{ kN}$$

$$W_u \text{ (or) } P_u = 1650 \text{ kN}$$

### Size of footing

$$A_f = \frac{\text{Total load}}{\text{Ultimate bearing capacity of soil}} = \frac{P_u}{SBC} = 7.33 \text{ m}^2$$

Adopt a footing of size  $6 \text{ m} \times 1.5 \text{ m}$ .

Adopt width of strap beam  $(b) = 400 \text{ mm}$

### Design of footing

$$\text{Soil Pressure } q_u = \frac{\text{Factored load}}{\text{Area of footing}} = \frac{1500 \text{ kN}}{\text{Area of footing}} = 166.6 \text{ kN/m}^2 < 225 \text{ kN/m}^2$$

$$\text{cantilever projection of footing } (L) = \frac{b_f - b_{\text{strap}}}{2} = 0.55 \text{ m}$$

$$\text{Ultimate design moment } M_u = \frac{q_u \cdot L^2}{2} = 25.2 \text{ kN.m}$$

### Eff. depth

(a) Moment consideration

$$d_f = \sqrt{\frac{M_u}{0.138 f_{ck} b}} = 96 \text{ mm} \approx 100 \text{ mm}$$

(b) shear consideration,  $d_f = 2$  times of moment consideration

$$d_f = 200 \text{ mm}$$

$$\text{Adopt } d_f = 250 \text{ mm} ; D = 300 \text{ mm}$$

### $A_{st}$

$$M_u = 0.87 f_y A_{st} d \left[ 1 - \frac{f_y A_{st}}{f_{ck} b d} \right]$$

$$A_{st} = 287 \text{ mm}^2$$

$$A_{st \text{ min}} = 0.12\% b D = 360 \text{ mm}^2$$

Main reinforcement

Adopt  $10 \text{ mm } \# @ 200 \text{ mm c/c}$  ( $A_{st} = 393 \text{ mm}^2$ )

### Check for Shear stress

shear stress at a distance = eff. depth

$$V_u = (L - d_f) q_u = 50 \text{ kN}$$

$$\tau_v = \frac{V_u}{b d} = 0.2 \text{ N/mm}^2$$

$$\frac{100 A_{st}}{b d} = 0.157\%$$

IS: 456-2000; P. NO: 73, Table: 19

$$\tau_c = 0.28 \text{ N/mm}^2$$

$$\text{Permissible shear stress} = k_s \cdot \tau_c = 1 \times 0.28 = 0.28 \text{ N/mm}^2$$

$k_s \cdot \tau_c > \tau_v$  ( $\therefore$  shear stress within the permissible limit)

## Design of strap beam

Factored load on beam  $W_u = (70 \times 6) = 252 \text{ kN/m}$

Neglecting small cantilever portion of the beam

$l = 4 \text{ m}$

$$M_u = \frac{W_u L^2}{8} = 506 \text{ kNm}$$

$$V_u = \frac{W_u L}{2} = 500 \text{ kN}$$

### depth of strap beam

Assuming  $\tau_c = 1.2 \text{ N/mm}^2$

$$d = \frac{V_u}{b \cdot \tau_c} = 1041 \text{ mm} \approx 1150 \text{ mm}$$

$D = 1200 \text{ mm}$

### A<sub>st</sub>

$$M_u = 0.87 f_y A_{st} d \left[ 1 - \frac{f_y A_{st}}{f_{ck} b d} \right]; A_{st} = 1270 \text{ mm}^2$$

Provide 4 - 22 mm # ( $A_{st} = 1520 \text{ mm}^2$ )

### Shear stress

$$\tau_v = \frac{V_u}{bd} = 1.09 \text{ N/mm}^2$$

$$\frac{100 A_{st}}{bd} = 0.33\% \therefore \tau_c = 0.4 \text{ N/mm}^2$$

(IS: 456-2000  
P. NO: 73  
Table: 19)

$\tau_v > \tau_c$  ( $\therefore$  shear reinforcement is necessary)

Shear reinforcement  $\rightarrow$  to resist balanced shear force

$$V_{us} = V_u - \tau_c \cdot b \cdot d = 316 \text{ kN}$$

use 8 mm # 4 legged stirrups

$$s_v = \frac{0.87 f_y A_{sv} d}{V_{us}} = 262 \text{ mm}$$

IS: 456-2000; P. NO: 73

Provide 4 legged 8 mm # stirrups at 250 mm/c

